I. Say whether this is true or false.

$$(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x + y = x)$$

False. One counterexample suffices to show that this is false; for example, if $x = 2$ and $y = 5$, then $2 + 5 \neq 2$.

II. Say whether this is true or false.

$$(\exists x \in \mathbb{Z})(\forall y \in \mathbb{Z})(x + y = x)$$

False. We need to argue for **every** $x \in \mathbb{Z}$ that there is some y such that $x + y \neq x$. Pick $y = 2$ for example. Then given any $x \in \mathbb{Z}$ we have $x + 2 \neq x$.

III. Say whether this is true or false.

$$(\forall x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x + y = x)$$

True. The same choice $y = 0$ works for all x. Given any $x \in \mathbb{Z}$ if we choose $y = 0$ we have $x + 0 = x$.

IV. Say whether this is true or false.

$$(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(x + y = x)$$

True. For example $x = 1$ and $y = 0$ gives $1 + 0 = 1$.

V. Say whether this is true or false.

$$(\exists y \in \mathbb{Z})(\forall x \in \mathbb{Z})(x + y = x)$$

True. This is the definition of y being an additive identity. So, we pick $y = 0$. Then, given any $x \in \mathbb{Z}$ we have $x + 0 = x$.