Monday 11/24/2014 Midterm III 9:30–10:20am

Name: ______________________ Student ID: ______________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	25	
Q2	25	
Q3	25	
Q4	25	
TOTAL	100	

Q1]...[25 points] How would you start to write down a proof that some set X is a subset of some other set Y? In other words, what is the key fact that you have to prove?

Suppose that A, B, C are sets. Write down a proof that if $A \subset B$, then $C - B \subset C - A$. Be sure to justify each step of your proof.

Q2]... [25 points] Say whether the following functions are *only injective*, *only surjective*, *bijective*, or *neither injective nor surjective*. It is important for you to give reasons for your answers.

1. $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} : (m, n) \mapsto 5^m 7^n$.

2. $g : \mathbb{Z} \to \mathbb{Z} : x \mapsto 3x - 4$.

3. $h : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto 3x + 4y$.

Q3]... [25 points] List the elements of the group G of symmetries of a square. How many elements does G have?

Find *two distinct subgroups* of $\mathrm{Perm}(\{1,2,3,4\})$ which are isomorphic to the group G above. Write down explicit bijections between G and these subgroups of $\mathrm{Perm}(\{1,2,3,4\})$. [Hint: Think about ways of labeling the vertices of the square with the numbers $1,2,3,4$.]

Q4]... [25 points] Say whether the following are True or False. Give a short reason (phrase, name of a theorem, example) for your answers.

1. Order((12345)) = 5.

2. $\mathbb{Z}_{10} - \{0\}$ is a group under multiplication.

3. The set of all subsets of a finite set A has $2^{|A|}$ elements.

4. If A has n elements, then the set of all *injective functions* from A to A has $n!$ elements.

5. $|A \cup B| = |A| + |B|$.

6. $\{\emptyset\} - \emptyset = \{\}$.