Comments about "proof strategy"

Several people tried to argue

If $4 \mid a$, then $4 \mid a^2$ (Q1)

or

If $11 \nmid a$, then $11 \nmid a^2$ (Q5)

by giving direct proofs of the contrapositive statements! These statements are (respectively)

If $4 \nmid a^2$, then $4 \nmid a$ (Q1)

or

If $11 \mid a^2$, then $11 \mid a$ (Q5)

You don't want to say: $11 \mid a^2 \Rightarrow a^2 \equiv 0 \bmod 11$

$\Rightarrow a = \sqrt{a^2} = \sqrt{0} \bmod 11$

etc without really understanding mod 11 arithmetic.

Likewise $a^2 \equiv 1 \Rightarrow a \equiv \sqrt{1} \equiv 1 \bmod 4$

require care....

[For example $2^2 \equiv 4 \equiv 0 \bmod 4$ so $\sqrt{0} \equiv 0 \pmod 4$ & $\sqrt{0} \equiv 2 \pmod 4$

↑ Here are several square roots of 0.]

another eg $\sqrt{1}$ has 4 answers (mod 12)

↝ 1, 11, 5, 7 all square to give 1 (mod 12)

On a more practical level, it is much easier to have some statement

$$a \equiv 0 \pmod 4$$

or

$$a \not\equiv 0 \pmod{11}$$

as hypothesis, & then to go squaring both sides to obtain information about a^2.

$$a^2 \equiv 0^2 \equiv 0 \pmod 4$$

or

$$a^2 \not\equiv 1^2, 2^2, 3^2, \ldots, 10^2 \pmod{11}$$

(10 cases)

ek...

The computations with multiplication (squaring) are very straightforward. The computations with $\sqrt{\ }$'s are not so clear cut, and require a deeper knowledge of the behavior of $\sqrt{\ }$'s in modular arithmetic.