Sp'16: MATH 2513–002	Discrete Mathematics	Noel Brady

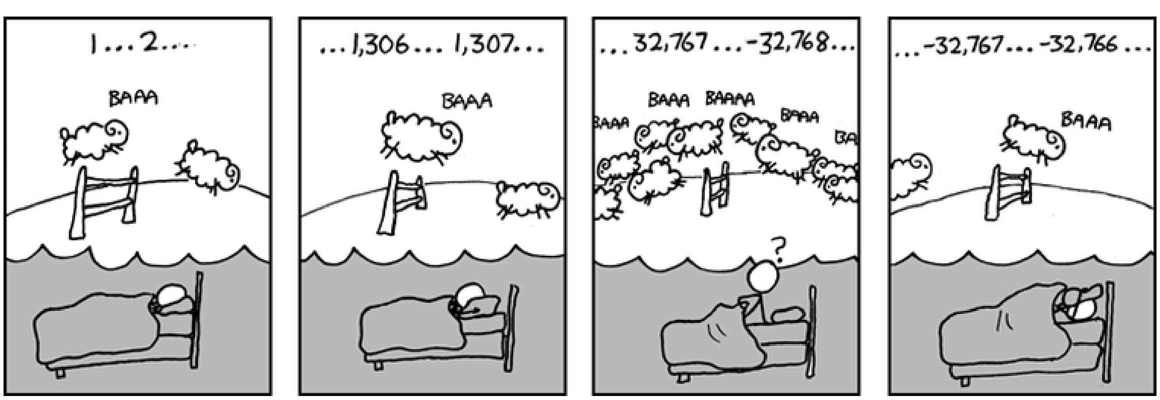
Friday 02/12/2016 — Midterm I — 50 mins

Name: ____________________ Student ID: ____________________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	20	
Q4	20	
Q5	20	
TOTAL	100	



Axioms of addition and multiplication of real numbers

The set of real numbers $\mathbb{R}$ is *closed* under addition (denoted by $+$) and multiplication (denoted by $\times$ or simply by juxtaposition) and satisfies:

1. Addition is *commutative.*
$$x + y = y + x$$
for all $x, y \in \mathbb{R}$.

2. Addition is *associative.*
$$(x + y) + z = x + (y + z)$$
for all $x, y, z \in \mathbb{R}$.

3. There is an *additive identity element.* There is a real number 0 with the property that
$$x + 0 = x$$
for all $x \in \mathbb{R}$.

4. Every real number has an *additive inverse.* Given any real number x, there is a real number $(-x)$ so that
$$x + (-x) = 0$$

5. Multiplication is *commutative.*
$$xy = yx$$
for all $x, y \in \mathbb{R}$.

6. Multiplication is *associative.*
$$(xy)z = x(yz)$$
for all $x, y, z \in \mathbb{R}$.

7. There is a *multiplicative identity element.* There is a real number 1 with the property that
$$x.1 = x$$
for all $x \in \mathbb{R}$. Furthermore, $1 \neq 0$.

8. Every non-zero real number has a *multiplicative inverse.* Given any real number $x \neq 0$, there is a real number $\frac{1}{x}$ so that
$$x\frac{1}{x} = 1$$

9. Multiplication *distributes* over addition
$$x(y + z) = xy + xz$$
for all $x, y, z \in \mathbb{R}$.

Q1]. . . [20 points] Consider the following statement about integers a.
Statement P: *If* $4 \mid a$*, then* $4 \mid a^2$*.*

1. Is statement P true or false?
   Please support your answer with a proof or counterexample.

2. Write down the converse of statement P.

3. Is the converse of P true or false?
   Please support your answer with a proof or counterexample.

4. Write down the contrapositive of P.

5. Is the contrapositive of P true or false?
   Please support your answer with a proof or counterexample.

Q2]. . . [20 points] Write down a truth table for the following compound statement

$$(\neg P) \wedge Q \wedge (\neg R)$$

Write down the negation of the statement above in a form that does not use the symbol $\wedge$.

Q3]... [20 points] Give a careful proof of the following proposition about real numbers x and y. If it helps, you may use the list of properties on the attached page and you may use the fact (proven in class) that the product of an arbitrary real number and 0 is equal to 0.

$$\textit{If } x \neq 0 \textit{ and } y \neq 0, \textit{ then } xy \neq 0.$$

Q4]. . . [20 points] Let $P(x, y, z)$ be the predicate $x + y = z$. Say which of the following quantified statements are true for the universal set $\mathbb{Z}$ of all integers. Give reasons to support your answers.

1. $(\exists x \in \mathbb{Z})(\exists y \in \mathbb{Z})(\exists z \in \mathbb{Z})P(x, y, z)$

2. $(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(\forall z \in \mathbb{Z})P(x, y, z)$

3. $(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})(\exists z \in \mathbb{Z})P(x, y, z)$

4. $(\exists z \in \mathbb{Z})(\forall x \in \mathbb{Z})(\forall y \in \mathbb{Z})P(x, y, z)$

Q5]... [20 points] Give a proof of the following statement about integers a:

$$\text{If } 11 \nmid a, \text{ then } 11 \nmid a^2$$

If it helps to speed up the proof, you can use congruences and their properties.

Use a divisibility test to check if the number $12,345,678$ divisible by 9? Give a clear statement of the test that you are using.