Miscellaneous Expressions and Results

- **Second Derivative Test.** Test depends on sign of D and of f_{xx}.

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2$$

- **Polar Coordinates.** $x = r\cos(\theta)$; $y = r\sin(\theta)$

$$dA = rdrd\theta$$

- **Cylindrical Coordinates.** $x = r\cos(\theta)$; $y = r\sin(\theta)$; $z = z$

$$dV = rdrd\theta dz$$

- **Spherical Coordinates.** $x = \rho\sin(\phi)\cos(\theta)$; $y = \rho\sin(\phi)\sin(\theta)$; $z = \rho\cos(\phi)$

$$dV = \rho^2\sin(\phi)d\rho d\phi d\theta$$

- **General Coordinates in 2-d.**

$$dA = |\frac{\partial(x,y)}{\partial(u,v)}|dudv$$

where

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

- **General Coordinates in 3-d.**

$$dV = |\frac{\partial(x,y,z)}{\partial(u,v,w)}|dudvdw$$

where

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix}$$

- **Surface Area.** Area element of the portion of the graph $z = f(x,y)$ which lies over the rectangle $dxdy$

$$dA = \sqrt{1 + f_x^2 + f_y^2}dxdy$$

- **Fundamental Theorem:**

$$\int_C (\nabla f)\cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

where the curve C is the curve given by $\mathbf{r}(t)$ where $a \le t \le b$.

- **Green's Theorem:** $\mathbf{F} = \langle P, Q\rangle$ is a vector field.

$$\oint_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$$

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_D \mathrm{curl}(\mathbf{F})\cdot\hat{\mathbf{k}}\,dA$$

where C is the positively oriented boundary of the 2-dimensional region D.

- **Stokes' Theorem:** $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field.

$$\oint_{\partial S} \mathbf{F} \cdot d\mathbf{r} \;=\; \iint_S \mathrm{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where ∂S is the positively oriented boundary of the oriented surface S in 3-dimensional space.

- **Divergence Theorem:** $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field.

$$\oiint_{\partial E} \mathbf{F} \cdot d\mathbf{S} \;=\; \iiint_E \mathrm{div}(\mathbf{F})\, dV$$

where ∂E is the positively oriented boundary of the 3-dimensional region E.

- **Surface area elements:**

$$d\mathbf{S} \;=\; \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}\, dudv$$

and

$$dS \;=\; \left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right| \, dudv$$

where the surface has parametric description $\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$.

- **Vector differential operator:**

$$\nabla \;=\; \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

can operate on functions (grad), and on vector fields either like a dot product (div) or like a cross product (curl).