Q4 Elliptical cylinder of height 2.

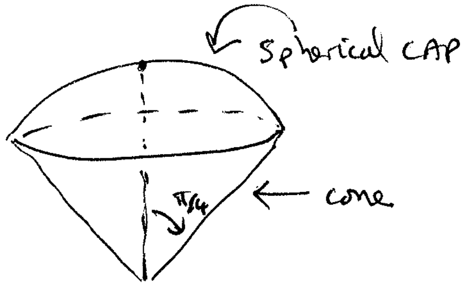
Cross section (parallel to xy-plane) is the ellipse.

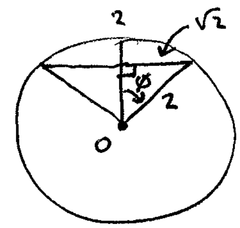
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

Q23

$$\left.\begin{aligned} z &= \sqrt{x^2+y^2} \\ x^2+y^2+z^2 &= 4 \end{aligned}\right\} \Rightarrow x^2+y^2+x^2+y^2 = 4$$

$x^2+y^2 = 2$
circle of radius $\sqrt{2}$.





$$\sin\phi = \frac{\sqrt{2}}{2}$$

$$\boxed{\phi = \pi/4}$$

$$\boxed{\begin{aligned} \vec{r}(\phi,\theta) &= \langle 2\sin\phi\cos\theta,\ 2\sin\phi\sin\theta,\ 2\cos\phi \rangle \\ & 0 \le \phi \le \pi/4 \\ & 0 \le \theta \le 2\pi \end{aligned}}$$

Q35

$$\vec{r}_u = \langle \cos v, \sin v, 0 \rangle$$

$$\vec{r}_v = \langle -u\sin v, u\cos v, 1 \rangle$$

$$\vec{r}(1,\pi/3) = \langle 1.\cos(\pi/3), 1.\sin(\pi/3), \pi/3 \rangle = \langle \tfrac{1}{2}, \tfrac{\sqrt{3}}{2}, \tfrac{\pi}{3} \rangle$$

$$\Rightarrow \vec{r}_u \times \vec{r}_v = \langle \sin v, -\cos v, u(\underbrace{\cos^2 v + \sin^2 v}_{1}) \rangle$$

$$\Rightarrow \vec{r}_u \times \vec{r}_v = \langle \sin(\pi/3), -\cos(\pi/3), 1 \rangle \quad \text{at } u=1,\ v=\pi/3$$

$$= \langle \tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}, 1 \rangle$$

Normal vector & Point

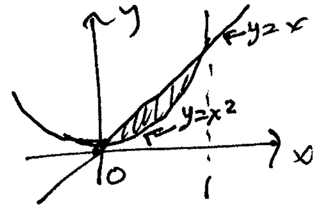
$$\boxed{\langle \tfrac{\sqrt{3}}{2}, -\tfrac{1}{2}, 1 \rangle \cdot \langle x - \tfrac{1}{2}, y - \tfrac{\sqrt{3}}{2}, z - \tfrac{\pi}{3} \rangle = 0}$$

Tangent plane

Q42

$$z = \sqrt{x^2+y^2}$$

Region in parameter space (xy-plane) is



$$\vec{r}(x,y) = \langle x, y, \sqrt{x^2+y^2} \rangle$$

$$\vec{r}_x = \left\langle 1, 0, \frac{x}{\sqrt{x^2+y^2}} \right\rangle$$

$$\vec{r}_y = \left\langle 0, 1, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$$

$$\vec{r}_x \times \vec{r}_y = \left\langle \frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1 \right\rangle$$

$$ds = |\vec{r}_x \times \vec{r}_y|\, dx\,dy = \sqrt{1 + \frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}}\; dx\,dy = \sqrt{2}\, dx\,dy$$

$$\text{Area} = \iint ds = \iint \sqrt{2}\, dx\,dy$$

$$= \sqrt{2} \int_0^1 \left(\int_{x^2}^{x} dy \right) dx$$

$$= \sqrt{2} \int_0^1 y \Big]_{x^2}^{x} dx = \sqrt{2} \int_0^1 x - x^2\, dx$$

$$= \sqrt{2} \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \sqrt{2}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\sqrt{2}}{6}$$

Q48

$$\vec{r}(u,v) = \langle u\cos v, u \sin v, v \rangle \qquad \begin{array}{l} 0 \le u \le 1 \\ 0 \le v \le \pi \end{array}$$

$$\vec{r}_u = \langle \cos v, \quad \sin v, \quad 0 \rangle$$

$$\vec{r}_v = \langle -u\sin v, \quad u\cos v, \quad 1 \rangle$$

$$\vec{r}_u \times \vec{r}_v = \langle \sin v, \quad -\cos v, \quad u\,(\cancel{\cos^2 v + \sin^2 v}^{\,1}) \rangle$$

$$dS = |\vec{r}_u \times \vec{r}_v|\, du\, dv$$

$$= \sqrt{\sin^2 v + \cos^2 v + u^2}\, du\, dv = \sqrt{1+u^2}\, du\, dv$$

$$\text{Area} = \int_0^{\pi} \left(\int_0^1 \sqrt{1+u^2}\, du \right) dv$$

(Tables) #21 at back of book.

$$= [v]_0^{\pi} \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\left(\ln\left(u + \sqrt{1+u^2}\right)\right) \right]_0^1$$

$$= \pi \left(\frac{\sqrt{2}}{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right) \right)$$