

Q1]... [10 points] Find and classify the critical points of the function

$$f(x, y) = x^4 + y^4 - 4xy$$

Q2]... [10 points] Use Lagrange multipliers to find the shortest distance from the origin to the surface $xyz^2 = 2$.

Q3]... [10 points] We are unable to anti-differentiate e^{-x^2} . However, we can still evaluate the double integral

$$\int_0^1 \int_y^1 e^{-x^2} dx dy$$

Show all the steps involved in evaluating this double integral.

Q4]... [10 points] Use double integrals to find the area of the portion of the conical surface $3z^2 = x^2 + y^2$ where $1 \leq z \leq 2$.

Q5]... [10 points] Consider the triple integral

$$\int_0^1 \int_z^1 \int_0^{x-z} f(x, y, z) dy dx dz$$

Sketch the projections of the region of integration on the three coordinate planes.

Rewrite the integral so that the outermost integral is with respect to x and the innermost integral is with respect to z .

Bonus]... Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 2$ of radius $\sqrt{2}$ which lies above the square with vertices $(\pm 1, \pm 1)$ in the xy -plane.



