Math 2443-006 Quiz VI – (1) Name:________________

Q1].. Find the area of the parametric surface

$$\mathbf{r}(u,v) = \langle v, u\sin v, u\cos v\rangle$$

where $0 \le u \le 1$ and $0 \le v \le \pi$.

$$\vec{r}_u = \langle 0, \sin v, \cos v\rangle$$

$$\vec{r}_v = \langle 1, u\cos v, -u\sin v\rangle$$

$$|\vec{r}_u \times \vec{r}_v| = |\langle -u, \cos v, -\sin v\rangle|$$

$$= \sqrt{u^2+1}$$

$$\boxed{\text{Area} = \int_0^{\pi}\int_0^1 \sqrt{u^2+1}\, du\, dv}$$

Math 2443-006 Quiz VI – (2) Name: SOLUTIONS

Q1].. Find the area of the portion of the paraboloid $x = y^2 + z^2$ which is contained inside of the cylinder $y^2 + z^2 = 4$.

$$\vec{r}(y,z) = \langle y^2 + z^2 , y , z \rangle , \quad 0 \le y^2+z^2 \le 4.$$

$$\vec{r}_y = \langle 2y, 1, 0 \rangle$$

$$\vec{r}_z = \langle 2z , 0, 1 \rangle$$

$$|\vec{r}_y \times \vec{r}_z| = |\langle 1, -2y , -2z \rangle|$$

$$= \sqrt{1 + 4(y^2+z^2)}$$

$$\boxed{\text{Area} = \iint_{y^2+z^2 \le 4} \sqrt{1 + 4(y^2+z^2)}\, dy\, dz \overset{\text{switch to polars}}{=} \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\, r\, dr\, d\theta}$$