Math 2443-006 Quiz V Name: SOLUTIONS

Q1].. State Green's Theorem (equation is sufficient).

$\vec{F} = \langle P, Q \rangle$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_{\text{Region}} (Q_x - P_y)\, dA$$

Region enclosed by simple, closed, positively oriented C.

Use Green's Theorem to evaluate the path integral

$$\int_C y^3\,dx - x^3\,dy$$

along the positively oriented circle $x^2 + y^2 = 4$.

$$\vec{F} = \langle y^3, -x^3 \rangle$$

$$Q_x - P_y = \frac{\partial}{\partial x}(-x^3) - \frac{\partial}{\partial y}(y^3) = -3(x^2+y^2)$$

$$\oint_C \vec{F}\cdot d\vec{r} = -3\iint_{\text{disk of radius 2 centered at }(0,0)} r^2 \, r\,dr\,d\theta = -3\int_0^2 r^3\,dr \int_0^{2\pi} d\theta$$

$$= -\frac{3}{4} r^4 \Big]_0^2 \; \theta \Big]_0^{2\pi}$$

$$= -24\pi$$