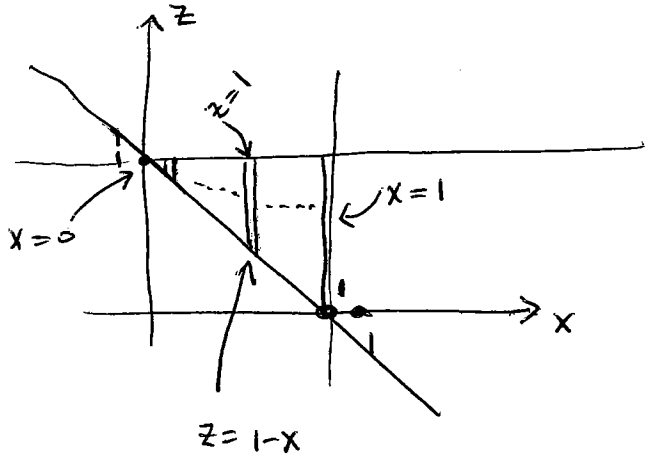
Math 2443-006 Quiz IV Name: CAREFUL SOLUTION

Q1].. Describe the region of integration of the triple integral

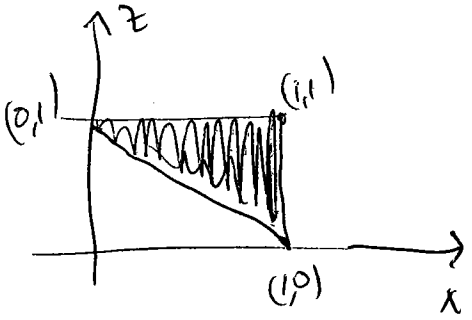
$$\int_0^1 \int_{1-x}^1 \int_0^{2-2z} dydzdx$$

In particular, you should sketch the projections of this region on the xy–, the yz– and the xz–planes. You should also sketch the 3–dimensional region itself or give a complete description.

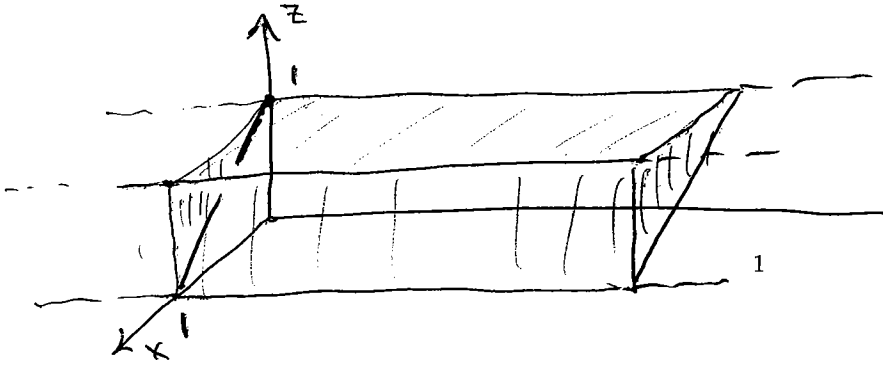
Step ① The outer $\iint$ is a $dzdx$ integral. $\Rightarrow$ we can easily find the projection onto the xz-plane.....



So Projection onto xz plane is the Right $\triangle$ shaded →



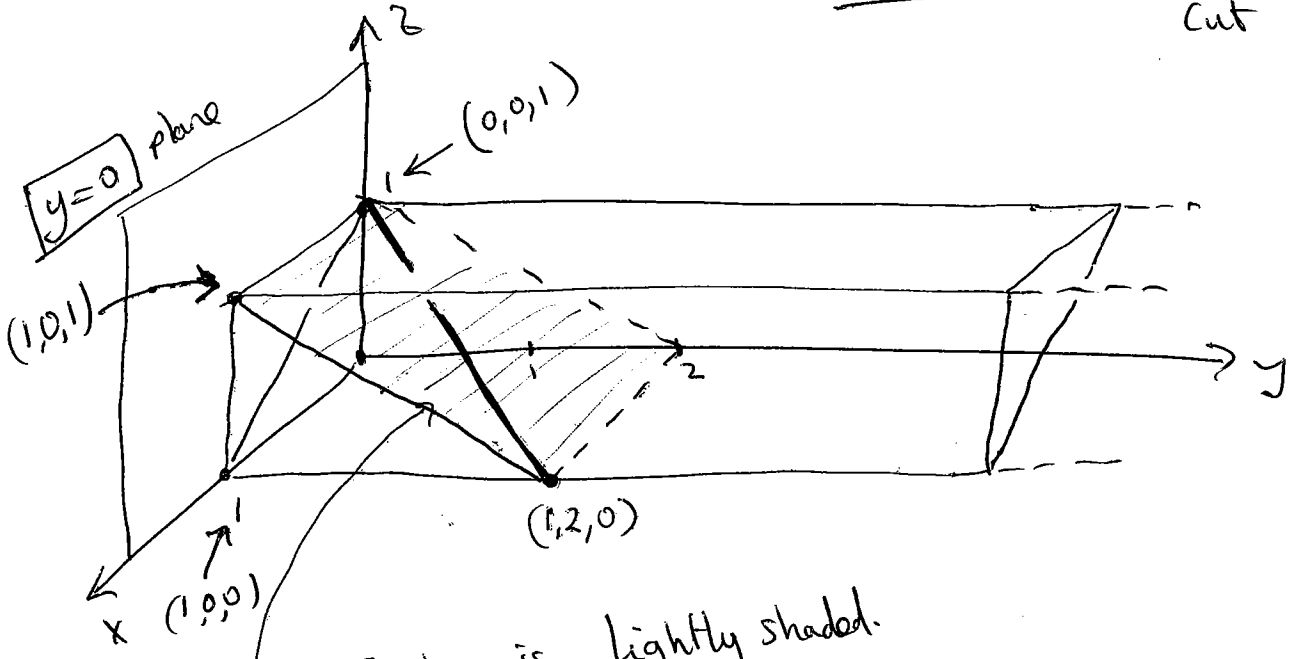
Step ② From Step ① we know that volume is obtained from the infinite triangular prism (parallel to y-axis) shown, by two planar slices.



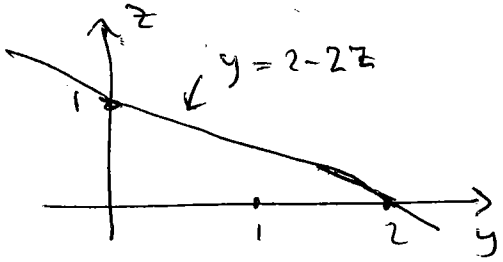
One is $\boxed{y=0}$,

2nd is $\boxed{y=2-2z}$.

Step ③ Draw in the two cut planes.



Now the 2nd plane $y = 2-2z$ meets (~~looks like~~) the line $y = 2-2z$ in the yz-plane & is parallel to the x-axis.

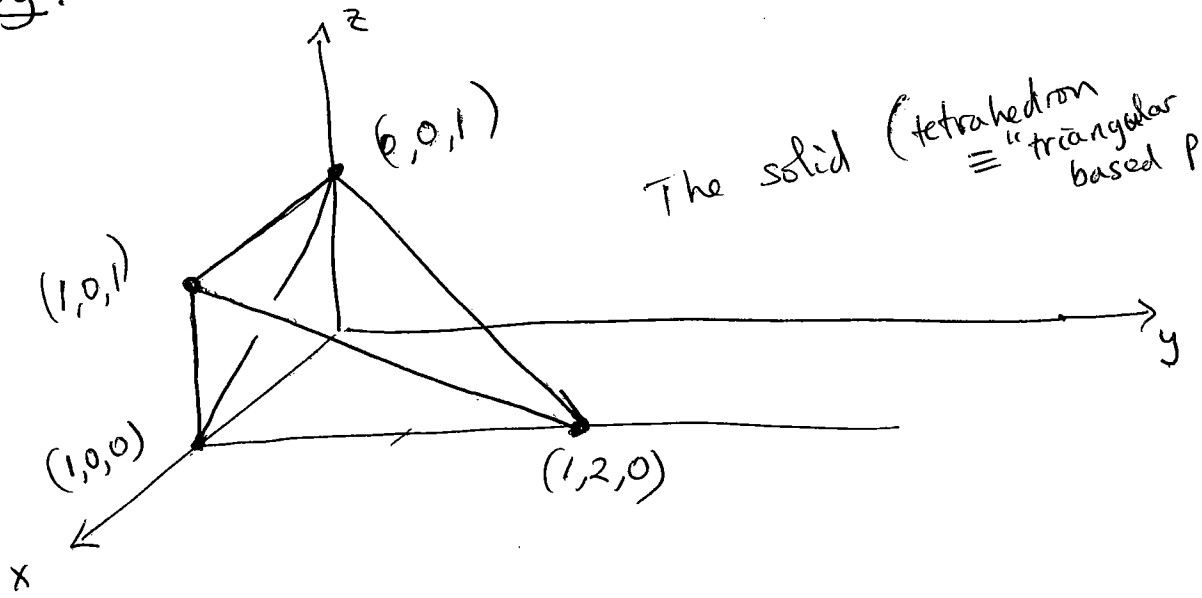


The plane $y = 2-z$ meets the plane $z = 1-x$ in the points $(0,0,1)$ and $(1,2,0)$

∴ These two planes meet in the line connecting $(0,0,1)$ & $(1,2,0)$.

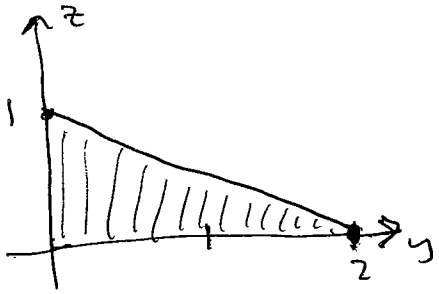
Ans: Region is the "tetrahedron" with corners $(0,0,1)$, $(1,2,0)$, $(1,0,0)$, $(1,0,1)$.

Summary!



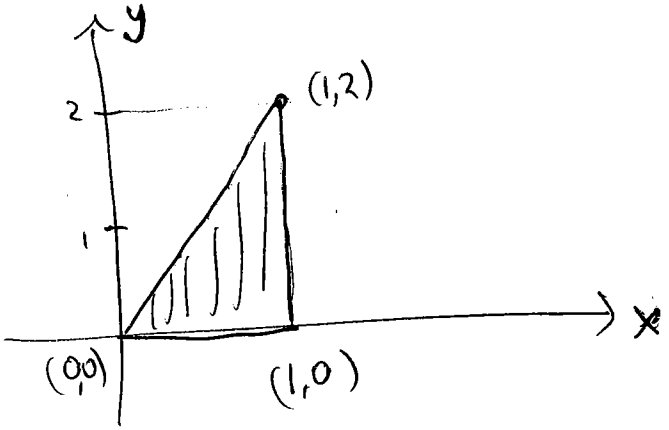
The solid (tetrahedron ≡ "triangular based pyramid")

Now it is easy to see the remaining two projections.



yz-plane projection

Triangle with vertices at (0,0) (0,1) (2,0)



xy-plane projection

Triangle with vertices at (0,0) (1,0) & (1,2)