

Q1].. Find the rate of change of the function $f(x, y, z) = (y^2 + \sin z)e^{-x}$ at the point $(0, 2, \pi)$ in the direction towards the point $(1, 1, 0)$.

$$f_x = -e^{-x} (y^2 + \sin z)$$

$$f_y = 2ye^{-x}$$

$$f_z = \cos z e^{-x}$$

$$\begin{aligned} \nabla f(0, 2, \pi) &= \langle -1(4 + \sin \pi), 4(1), \cos \pi(1) \rangle \\ &= \langle -4, 4, -1 \rangle \end{aligned}$$

Diagram showing a vector \vec{u} pointing from the point $(0, 2, \pi)$ to the point $(1, 1, 0)$. The vector is labeled $\vec{u} = \langle 1-0, 1-2, 0-\pi \rangle = \langle 1, -1, -\pi \rangle$.

$$\begin{aligned} \hat{u} &= \frac{\langle 1, -1, -\pi \rangle}{|\langle 1, -1, -\pi \rangle|} \\ &= \frac{\langle 1, -1, -\pi \rangle}{\sqrt{\pi^2 + 2}} \end{aligned}$$

$$D_{\hat{u}} f(0, 2, \pi) = \langle -4, 4, -1 \rangle \cdot \frac{\langle 1, -1, -\pi \rangle}{\sqrt{\pi^2 + 2}}$$

$$= \frac{-8 + \pi}{\sqrt{\pi^2 + 2}}$$