Homework #2

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8) $f(t) = (2+t^4)^5$ and $g(x) = \int_1^x (2+t^4)^5\,dt$, so $g'(x) = f(x) = (2+x^4)^5$

16) Let $u = \cos x$. Then $\frac{du}{dx} = -\sin x$. Also, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, So

$$y' = \frac{d}{dx}\int_1^{\cos x} (t+\sin t)\,dt = \frac{d}{du}\int_1^{u} (t+\sin t)\,dt \cdot \frac{du}{dx}$$

$$= (u+\sin u) \cdot (-\sin x)$$

$$= -\sin x\left[\cos x + \sin(\cos x)\right].$$

18) Some idea with sixteen; $u = \frac{1}{x^2}$, $\frac{du}{dx} = \frac{-2}{x^3}$. Also $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

So $y' = \frac{d}{dx}\int_{1/x^2}^{0} \sin^3 t\,dt = \frac{d}{du}\int_u^0 \sin^3 t\,dt \cdot \frac{du}{dx} = -\frac{d}{du}\int_0^u \sin^3 t\,dt \cdot \frac{du}{dx} = -\sin^3 u \cdot \left(\frac{-2}{x^3}\right)$

$$= \frac{2\sin^3\left(1/x^2\right)}{x^3}$$

22) $\int_0^4 (1+3y-y^2)\,dy = \left[y + \frac{3y^2}{2} - \frac{y^3}{3}\right]_0^4 = \frac{20}{3}$

26) $\int_{-2}^{3} x^{-5}\,dx$ does not exist because $f(x) = x^{-5}$ has an infinite discontinuity at $x=0$

hence f is discontinuous on $[-2,3]$.

28) $\int_{\pi}^{2\pi} \cos\theta\,d\theta = [\sin\theta]_{\pi}^{2\pi} = \sin 2\pi - \sin\pi = 0$

32) $\int_0^1 (3+x\sqrt{x})\,dx = \int_0^1 (3+x^{3/2})\,dx = \left[3x + \frac{2}{5}x^{5/2}\right]_0^1 = \left(3+\frac{2}{5}\right) - 0 = \frac{17}{5}$

34) $\int_0^{\pi/6} \csc\theta \cdot \cot\theta \, d\theta$ does not exist because the function $f(\theta) = \csc\theta \cdot \cot\theta$ has an infinite discontinuity at $\theta = 0$, hence f is discontinuous on $[0, \frac{\pi}{6}]$.

48) For the curve to be concave upward, we must have $y'' > 0$ hence

$$y = \int_0^x \frac{1}{1+t+t^2} \, dt$$

$$\Rightarrow y' = \frac{1}{1+x+x^2}$$ by the Fundamental Theorem of Calculus.

$$\Rightarrow y'' = \frac{-(1+2x)}{(1+x+x^2)^2}$$

So $y'' > 0$ when $-(1+2x) > 0$ hence $x < -\frac{1}{2}$. Thus the curve is concave upward when $x \in (-\infty, -\frac{1}{2}]$.

53) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = \lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 = \int_0^1 x^3 \, dx = \frac{1}{4}$

54) $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right) = \lim_{n\to\infty} \frac{1-0}{n} \sum_{i=1}^{n} \sqrt{\frac{i}{n}} = \int_0^1 \sqrt{x} \, dx = \left[\frac{2}{3} x^{3/2}\right]_0^1 = \frac{2}{3}$

56) $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t)\,dt = \frac{d}{dx}\left[\int_{g(x)}^{a} f(t)\,dt + \int_a^{h(x)} f(t)\,dt\right]$ (where $a \in$ (Domain of f))

$$= \frac{d}{dx}\left[-\int_a^{g(x)} f(t)\,dt + \int_a^{h(x)} f(t)\,dt\right]$$

$= -f(g(x)) \cdot g'(x) + f(h(x)) \cdot h'(x)$ by Fund. Thm. of Calc.

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6) $\int \sqrt[3]{x}\,dx = \int x^{1/3}dx = \frac{x^{4/3}}{\frac{4}{3}} + c = \frac{3}{4}\cdot x^{4/3} + c$

12) $\int(\sin\theta + 3\cos\theta)d\theta = \int \sin\theta\, d\theta + 3\int\cos\theta\, d\theta = -\cos\theta + 3\sin\theta + c$

24) $\int_0^9 \sqrt{2t}\,dt = \int_0^9 \sqrt{2}.\sqrt{t}\,dt = \sqrt{2}.\int_0^9 t^{1/2}dt = \sqrt{2}.\left[\frac{2}{3}t^{3/2}\right]_0^9 = \sqrt{2}.\frac{2}{3}.27 - 0 = 18\sqrt{2}.$

32) $\int_{\pi/4}^{\pi/3} \sec\theta\tan\theta\, d\theta = [\sec\theta]_{\pi/4}^{\pi/3} = \sec\frac{\pi}{3} - \sec\frac{\pi}{4} = 2 - \sqrt{2}.$

40) $\int_0^{3\pi/2} |\sin x|\,dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{3\pi/2} (-\sin x)\,dx = [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{\frac{3\pi}{2}} = [1-(-1)] + [0-(-1)]$
$= 2 + 1 = 3.$

48) By the Net Change theorem

$$\int_0^{15} n'(t)dt = n(15) - n(0) = n(15) - 100$$

represents the increase in the bee population in 15 weeks. So $100 + \int_0^{15} n'(t)dt$ $= n(15)$ represents the total bee population after 15 weeks.

49) $\int_{1000}^{5000} R'(x)dx = R(5000) - R(1000)$ by the Net change thm.

It represents the increase in revenue when production is increased from 1000 units to 5000 units

50) The slope of the trail is the rate of change of the Evelation, so $f(x) = E'(x)$. By the Net Change theorem

$$\int_3^5 f(x)dx = \int_3^5 E'(x)\,dx = E(5) - E(3) \quad \text{by Fund. Thm. of Calc.}$$

This is the change in the elevation E between $x=3$ miles and $x=5$ miles from the start of the trail.

54) a) Displacement $= \int_1^6 (t^2 - 2t - 8)dt = \left[\frac{1}{3}t^3 - t^2 - 8t\right]_1^6 = -\frac{10}{3}$ m

b) Distance traveled $= \int_1^6 |t^2 - 2t - 8|dt = \int_1^4 (-t^2 + 2t + 8)dt + \int_4^6 (t^2 - 2t - 8)dt.$

$= \frac{98}{3}$ m

56) a) $v'(t) = a(t) = 2t + 3 \Rightarrow v(t) = t^2 + 3t + c \Rightarrow v(0) = c = -4 \Rightarrow v(t) = t^2 + 3t - 4$

b) Distance Traveled $= \int_0^3 |t^2 + 3t - 4|\,dt$

$= \int_0^1 (-t^2 - 3t + 4)dt + \int_1^3 (t^2 + 3t - 4)dt$

$= \frac{89}{6}$ m.

57) Since $m'(x) = \rho(x)$, $m = \int_0^4 \rho(x)dx = \int_0^4 (9 + 2\sqrt{x})dx = 46\frac{2}{3}$ kg.