# Hmk #1

1)

$$(n+1)^3 - \cancel{n^3} = 3n^2 + 3n + 1$$

$$\cancel{n^3} - \cancel{(n-1)^3} = 3(n-1)^2 + 3(n-1) + 1$$

$$\cancel{(n-1)^3} - \cancel{(n-2)^3} = 3(n-2)^2 + 3(n-2) + 1$$

$$\vdots$$

$$\cancel{3^3} - \cancel{2^3} = 3(2^2) + 3.(2) + 1$$

$$\cancel{2^3} - 1^3 = 3(1)^3 + 3.(1) + 1$$

$$+ \quad (n+1)^3 - 1^3 = 3\sum_{i=1}^{n} i^2 + 3\left(\sum_{i=1}^{n} i\right) + n$$

We know that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ hence

$$(n+1)^3 - 1^3 = 3\sum_{i=1}^{n} i^2 + \frac{3n(n+1)}{2} + n$$

$$\Rightarrow [(n+1)-1][(n+1)^2 + (n+1) + 1^2] = 3\sum_{i=1}^{n} i^2 + \frac{3n^2+5n}{2}$$

$$\Rightarrow n.(n^2+2n+1+n+1+1) = 3\sum_{i=1}^{n} i^2 + \frac{3n^2+5n}{2}$$

$$\Rightarrow n^3 + 2n^2 + n + n^2 + 2n - \frac{3n^2+5n}{2} = 3\sum_{i=1}^{n} i^2$$

$$\Rightarrow n^3 + 3n^2 + 3n - \frac{3n^2+5n}{2} = 3\sum_{i=1}^{n} i^2$$

$$\Rightarrow \frac{2n^3 + 6n^2 + 6n - 3n^2 - 5n}{2} = 3\sum_{i=1}^{n} i^2$$

$$\Rightarrow \frac{2n^3 + 3n^2 + n}{6} = \sum_{i=1}^{n} i^2$$

$$\Rightarrow \frac{n(2n^2+3n+1)}{6} = \sum_{i=1}^{n} i^2$$

$$\Rightarrow \frac{n(n+1)(2n+1)}{6} = \sum_{i=1}^{m} i^2$$

2) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

3)
$$
\begin{aligned}
(n+1)^4 - n^4 &= 4n^3 + 6n^2 + 4n + 1 \\
n^4 - (n-1)^4 &= 4(n-1)^3 + 6(n-1)^2 + 4(n-1) + 1 \\
&\vdots \\
+ \quad 2^4 - 1^4 &= 4.1^3 + 6.1^2 + 4.1 + 1 \\
\hline
(n+1)^4 - 1^4 &= 4\sum_{i=1}^{n} i^3 + 6\sum_{i=1}^{n} i^2 + 4\sum_{i=1}^{n} i + n
\end{aligned}
$$

We know that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ hence

$$(n+1)^4 - 1 = 4\sum_{i=1}^{n} i^3 + \cancel{6}\frac{n.(n+1)(2n+1)}{\cancel{6}} + \overset{2}{\cancel{4}}\frac{n(n+1)}{\cancel{2}} + n$$

$$[(n+1)^2 - 1].[(n+1)^2 + 1] - n(n+1)(2n+1) - 2n(n+1) - n = 4\sum_{i=1}^{n} i^3$$

$$[(n+1) - 1][n+1+1][n^2 + 2n + 2] - n(n+1)(2n+1) - 2n(n+1) - n = 4\sum_{i=1}^{n} i^3$$

$$n.(n+2).(n^2 + 2n + 2) - n(n+1)(2n+1) - 2n(n+1) - n = 4\sum_{i=1}^{n} i^3$$

$$n[(n+2)(n^2 + 2n + 2) - (n+1)(2n+1) - 2(n+1) - 1] = 4\sum_{i=1}^{n} i^3$$

$$n(n^3 + \cancel{2n^2} + \cancel{2n} + 2n^2 + 4n + 4 - \cancel{2n^2} - n - \cancel{2n} - 1 - 2n - 2 - 1] = 4\sum_{i=1}^{n} i^3$$

$$\frac{n(n^3 + 2n^2 + n)}{4} = \sum_{i=1}^{n} i^3$$

$$\Rightarrow \frac{n.n(n+1)^2}{4} = \sum_{i=1}^{n} i^3$$

$$\Rightarrow \left[\frac{n(n+1)}{2}\right]^2 = \sum_{i=1}^{n} i^3$$

4. Find the area under the graph $y=x^3$ between $x=0$ and $x=1$.

by using limits of Riemann sums.

<u>solution</u>: $\int_0^1 x^3 dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ where $x_i = a + i.\Delta x$ and $\Delta x = \frac{b-a}{n}$

$$x_i = 0 + i.\Delta x \text{ and } \Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x_i = i.\frac{1}{n} = \frac{i}{n} \text{ and } \Delta x = \frac{1}{n}$$

hence

$$\int_0^1 x^3 dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3} \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4} = \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3$$

$$= \lim_{n\to\infty} \frac{1}{n^4} \cdot \left[\frac{n(n+1)}{4}\right]^2$$

$$= \lim_{n\to\infty} \frac{1.n^4 + 2n^3 + n^2}{4n^4} = \frac{1}{4}$$

5. Write out the following without using sommation notain:

a) $\sum_{i=1}^{5} \frac{1}{i+3} = \frac{1}{1+3} + \frac{1}{2+3} + \frac{1}{3+3} + \frac{1}{4+3} + \frac{1}{5+3}$

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

b) $\sum_{i=4}^{8} \frac{1}{i} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

c) $\sum_{i=10}^{14} \frac{1}{i-6} = \frac{1}{10-6} + \frac{1}{11-6} + \frac{1}{12-6} + \frac{1}{13-6} + \frac{1}{14-6}$

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}$

6). a) $\sum_{i=1}^{n} \left(1+\frac{i}{n}\right)\left(\frac{1}{n}\right) \longrightarrow f(x_i) = x_i$ for $[1,2]$ where $\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$f(x_i) = -1 + x_i$ for $[2,3]$ where $\Delta x = \frac{3-2}{n} = \frac{1}{n}$

$f(x_i) = -2 + x_i$ for $[3,4]$ where $\Delta x = \frac{4-3}{n} = \frac{1}{n}$

$\vdots$

$f(x_i) = -k + 1 + x_i$ for $[k, k+1]$ where $\Delta x = \frac{k+1-k}{n} = \frac{1}{n}$

hence for all $k \in \mathbb{R}$, $f(x) = -k+1+x$ for the interval $[k, k+1]$

b) $\sum_{i=1}^{n} \left(\frac{1}{n\left(2+\frac{i}{n}\right)^2}\right) = \sum_{i=1}^{n} \frac{1}{\left(2+\frac{i}{n}\right)^2} \cdot \frac{1}{n} \rightarrow f(x_i) = \frac{1}{(2+x_i)^2}$ for $[0,1]$ where $\Delta x = \frac{1}{n}$

$f(x_i) = \frac{1}{(2-1+x_i)^2}$ for $[1,2]$ where $\Delta x = \frac{1}{n}$

$f(x_i) = \frac{1}{(2-2+x_i)^2}$ for $[2,3]$ where $\Delta x = \frac{1}{n}$

hence for all $k \in \mathbb{R}$, $f(x) = \frac{1}{(2-k+x)^2}$ for $[k, k+1]$ on the interval $[k, k+1]$.