Honors Calculus II [2423-001] Quiz I

Q1]...[10 points] Write down the expressions (formulas) for

\[ \sum_{i=1}^{n} i \quad \text{and for} \quad \sum_{i=1}^{n} i^2 \]

Solutions:

\[ \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \]

and

\[ \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \]

Compute the definite integral below using limits of Riemann sums.

\[ \int_{1}^{2} x^2 - x \, dx \]

Solution: Using \( n \) subintervals of equal width, we see that the widths are \( \Delta x_i = (2 - 1)/n = 1/n \), and that the right-hand endpoints of the intervals are \( x_i = 1 + i/n \) for \( 1 \leq i \leq n \).

Thus the Riemann sums (using right-hand endpoints) become

\[
\sum_{i=1}^{n} \left( (1 + i/n)^2 - (1 + i/n) \right) \Delta x_i = \sum_{i=1}^{n} \left( 1 + 2i/n + i^2/n^2 - 1 - i/n \right) (1/n)
\]

\[
= \left( 1/n^2 \right) \sum_{i=1}^{n} (i + i^2/n)
\]

\[
= \left( 1/n^2 \right) [n(n+1)/2 + n(n+1)(2n+1)/6n]
\]

\[
= \frac{3n^2 + 3n + 2n^2 + 3n + 1}{6n^2}
\]

\[
= \frac{5n^2 + 6n + 1}{6n^2}
\]

which tends to \( 5/6 \) as \( n \to \infty \).