

Friday 09/21/2007

Midterm I

50 minutes

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.
4. No calculators, no notes, no books.

Question	Points	Your Score
Q1	15	
Q2	15	
Q3	8	
Q4	12	
TOTAL	50	

Q1]... [15 points] For each of the following, say if the statement is true or false.

(a) If $f(x)$ and $g(x)$ each have second derivatives, then

$$\frac{d^2(fg)}{dx^2} = \frac{d^2f}{dx^2}g + f\frac{d^2g}{dx^2}$$

(b) If the position of a particle at time t is given by $x(t) = t^3 - 3t^2$, then the particle is decelerating (slowing down) during the interval from time $t = 0$ until time $t = 1$.

(c) If $f(x)$ is differentiable at the point a then

$$\lim_{h \rightarrow 0} \frac{f(a) - f(a-h)}{h} = -f'(a)$$

(d) The piecewise defined function y is continuous at 0

$$y = \begin{cases} x \sin(1/x) & \text{when } x < 0 \\ 0 & \text{when } x = 0 \\ x^2 \cos(1/x) & \text{when } x > 0 \end{cases}$$

(e)

$$\lim_{x \rightarrow 3} \frac{x^{10} - 3^{10}}{x - 3} = 10(3^9)$$

Q2]... [15 points] Write down the values of the following two limits (you do **not** have to give proofs).

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x}$$

Write out the angle addition formula for the cosine function.

$$\cos(A + B) =$$

Compute the derivative of $\cos(x)$ at the point a using the limit of the difference quotient definition of derivative. Show all your work.

Q3... [8 points] Verify that the graphs of $y = x^2$ and $y = \frac{1}{\sqrt{x}}$ intersect at the point $(1, 1)$.

Show that these graphs are perpendicular at the intersection point $(1, 1)$; that is, show that their tangent lines at the point $(1, 1)$ are perpendicular.

Q4]... [12 points] Compute the derivatives y' of the following functions. Write down the names of the differentiation rules that you used in each case.

$$y = \frac{(\sin(x) + 4x + 3)}{(x^8 - 5x)}$$

$$y = (\sqrt{x} + x + 7)(x^8 - 5x + 3)$$