

Q1].. The equation

$$x^2 + 4y^2 - 4x + 8y = 1$$

represents an ellipse in the plane.

Draw this ellipse, indicating its center, foci and points of intersection with horizontal and vertical lines through its center.

$$x^2 - 4x + 4 + 4(y^2 + 2y) = 1$$

$$x^2 - 4x + 4 + 4(y^2 + 2y + 1) = 1 + 4 + 4(1)$$

$$(x-2)^2 + 4(y+1)^2 = 9$$

$$\left(\frac{x-2}{3}\right)^2 + \left(\frac{y+1}{3/2}\right)^2 = 1 \quad \text{Ellipse}$$

$$\text{Center} = (2, -1)$$

$$c = \sqrt{a^2 - b^2} = \sqrt{3^2 - \left(\frac{3}{2}\right)^2}$$

$$= \sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2}$$

$$\text{Vertices} = \left(2 + \frac{3\sqrt{3}}{2}, -1\right) \quad \left(2 - \frac{3\sqrt{3}}{2}, -1\right)$$

$$\text{Horizontal line intersections: } (2, -1) \pm (3, 0) = (-1, -1) \text{ and } (5, -1)$$

$$\text{Vertical line intersections: } (2, -1) \pm (0, 3/2) = \left(2, \frac{1}{2}\right) \text{ and } \left(2, -\frac{5}{2}\right)$$

