Q1... [13 points] Find the coordinates of the points on the ellipse

\[ x^2 + \frac{y^2}{4} = 1 \]

which are farthest from the point (2, 0).

\[
\begin{align*}
(\text{dist})^2 &= (x-2)^2 + y^2 \\
f(x) &= (x-2)^2 + 4 - 4x^2 \\
f'(x) &= 2(x-2) - 8x \\
&= -4 - 6x \\
f'(x) &= 0 \quad x = \frac{-2}{3} \\
y &= \pm \frac{2\sqrt{5}}{3}
\end{align*}
\]

\[ \left( \frac{-2}{3}, \pm \frac{2\sqrt{5}}{3} \right) \]
Q2]... [12 points] Express the following limit as a definite integral

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \left(\frac{2i}{n}\right)^2 \cos \left(\frac{2i}{n}\right)^3
\]

\[
\Delta x \rightarrow \frac{2-0}{n} \Rightarrow \frac{2}{n}
\]

\[
\left[0, 2\right] \text{ interval}
\]

\[
\int_{0}^{2} x^2 \cos(x^3) \, dx
\]

Using whatever method you like, evaluate the definite integral that you obtained above.

\[
\text{Subst.} \quad u = x^3
\]

\[
\begin{align*}
    x = 0 & \Rightarrow u = 0^3 = 0 \\
    x = 2 & \Rightarrow u = 2^3 = 8
\end{align*}
\]

\[
du = 3x^2 \, dx
\]

\[
\int_{0}^{8} \cos(u) \, \frac{du}{3} = \frac{\sin(u)}{3} \bigg|_{0}^{8}
\]

\[
= \frac{\sin(8)}{3}
\]
Q3]... [12 points] Compute the area of the region between the graph of $y = \sqrt{x}$ and the graph of $y = x^3$.

Intersection of 2 graphs:

$x^3 = \sqrt{x}$
$x^6 = x$
$(x^5 - 1)x = 0$
$x = 0, x^5 = 1$
$x = 0, x = 1$

$$A = \int_{0}^{1} x^6 - x^3 \, dx$$

$$= \left[ \frac{2}{3} x^{\frac{3}{2}} - \frac{x^4}{4} \right]_{0}^{1}$$

$$= \frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$$
Q4... [13 points] Write down a general expression for computing the volume of a 3-dimensional shape by slicing perpendicular to an axis.

\[ V = \int_a^b A(x) \, dx \]

\[ \text{Area of cross sectional slice} \perp \text{to axis at } x, \]

\[ [a, b] = \text{projection of object onto axis}. \]

Compute the volume of a spherical cap of height \( R/2 \) and sphere radius \( R \).

\[
\text{Cross - Section :}
\]

\[ x^2 + (r_{ad})^2 = R^2 \]

\[ (r_{ad})^2 = R^2 - x^2 \]

\[ V = \int_{R/2}^R \pi (R^2 - x^2) \, dx \]

\[ = \pi \left[ R^2 x - \frac{x^3}{3} \right]_{R/2}^R \]

\[ = \pi \left( \frac{2R^3}{3} - \frac{R^3}{2} + \frac{R^3}{24} \right) \]

\[ = \frac{\pi R^3}{24} \left( 8(2) - 12 + 1 \right) \]

\[ = \frac{5\pi R^3}{24} \]