Cool Fact: Let $l$ denote 'reflection in the line $l$' and $M$ denote 'reflection in the line $M$'.

If lines $l$, $M$ meet in a point $O$, and make an angle of $\Theta$, then:

$$\text{mol} = \text{Rotation about } O, \text{ through an angle of } 2\Theta, \text{ in direction from } l \text{ to } M \text{ (across acute angle).}$$

Example:

$$\text{mol} = \text{counter clockwise rotation about } O \text{ through } \angle(30) = 60^\circ,$$

$$\text{lom} = \text{clockwise rotation about } O \text{ through } \angle(30) = 60^\circ.$$ 

Note: $(\text{lom})o(\text{mol}) = l = \text{mol} \Rightarrow 1$

$$= \sqrt{1} = 1 \text{ makes sense, since clockwise }$$

$$= 1 \text{ + counter clockwise rotations out!}$$
\[ R = \text{counter clockwise rotation about 0 through } \pi/2. \]

\[ R^2 = R \circ R \quad R^3 = R \circ R \circ R \quad R^4 = R \circ R \circ R \circ R = 1. \]

\[
\begin{array}{cccccccc}
\text{composition symbol} & \to & 0 & 1 & R & R^2 & R^3 & p & q & r & s \\
1 & 1 & R & R^2 & R^3 & p & q & r & s \\
R & R & R^2 & R^3 & 1 & s & p & q & r \\
R^2 & R^2 & R^3 & 1 & R & s & p & q & r \\
R^3 & R^3 & 1 & R & R^2 & q & r & s & p \\
p & p & q & r & s & 1 & R & R^2 & R^3 \\
p & p & q & r & s & 1 & R & R^2 & R^3 \\
r & r & s & p & q & R^3 & 1 & R & R^2 \\
r & r & s & p & q & R^3 & 1 & R & R^2 \\
s & s & p & q & r & R R^2 & R^3 & 1 & R \\
s & s & p & q & r & R R^2 & R^3 & 1 & R \\
\end{array}
\]
IDEA

- Top left square is easy!!
- Bottom right square is OK!! (just use "cool fact" over & over again)
- For remaining squares use algebra:

\[
R \circ p = (s \circ p) \circ p = s(\circ p) = s \circ 1 = s
\]

\[
and \quad p \circ R = p \circ (p \circ q) = (p \circ p) \circ q = 1 \circ q = q.
\]

There are very fast ways of reading off & filling in values — via rows/_columns of original table.

Think about it!!
$R = \text{rotation counter clockwise about } O \text{ through } \frac{2\pi}{5}$

$R^2 = R \circ R, \; R^3 = R \circ R^2, \; R^4 = R^3 \circ R, \; R^5 = R^4 \circ R = 1$. 

There are 5 reflections in lines $l, p, q, r, s, t$ and 5 rotations about $O: R, R^2, R^3, R^4, R^5 = 1$. 

Comp table.
Some patterns should emerge as you fill in the squares 66.
Answer: No reflection symmetry. Just rotations about 0° through \( \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5} \).

\[
\begin{align*}
R & \leftrightarrow (1 2 3 4 5) \\
R^2 & \leftrightarrow (1 3 5 2 4) \\
R^3 & \leftrightarrow (1 4 2 5 3) \\
R^4 & \leftrightarrow (1 5 4 3 2) \\
1 = R^5 & \leftrightarrow 1
\end{align*}
\]

They form a (cyclic) subset \( \text{perm}\{1, -, 5\} \) which are closed under composition.

\[
\begin{align*}
\{1, R, R^2, R^3, R^4\} & \subseteq \{1, R, R^2, R^3, R^4, R^5\} \subseteq \text{perm}\{1, 2, 3, 4, 5\} \uparrow \\
\text{Has 5 elements} & \uparrow \downarrow \text{Has 10 elements} \\
\text{Has 120 elements} & \\
\text{Note: 5 divides 10, & 10 divides 120.}
\end{align*}
\]