Review for Midterm 2

Extra office hour: Monday 9:30–11:00am. Exam covers 3.4-6.2

On Sequences and series (Chapter 3). Bolzano-Weierstrass Theorem, Cauchy criterion, convergence or divergence for sequences and series.

Exercise 1:
(a). If \( \{x_n\} \) is a monotone increasing sequence that is bounded above by \( M \). Prove that \( \lim x_n \) exists.
(b). Give examples to show that a bounded sequence may not be a Cauchy sequence. Will a Cauchy sequence be bounded?

Exercise 2: Let \( \sum_{n=1}^{\infty} a_n \) be a convergent series and \( a_n > 0 \). If \( b_k = \frac{a_1 + a_2 + \cdots + a_k}{k} \), then
(a). \( \lim b_k = 0 \).
(b). Will \( \sum_{k=1}^{\infty} b_k \) be a convergent or a divergent series?

Limits for functions (Chapter 4). Definition, the link between limits of functions and limits of sequences.

Exercise 3: Show that \( \lim_{x \to 0} \sin \frac{1}{x} \) does not exist.

Continuity for functions (Chapter 5).
Basic algebraic properties: Algebraic operations and composition functions.

Exercise 4. (a) Find two functions defined in \( R \) such that both of them are discontinuous everywhere, but \( f + g \) is continuous everywhere.
(b). Find two functions defined in \( R \) such that both of them are discontinuous everywhere, but \( g \circ f \) is continuous everywhere.
Continuous functions on closed interval: extremum, intermediate value theorem, uniformly continuous.

Exercise 5:
(a). Give an example (and prove your statement) that a continuous function \( f(x) \) in an open interval \((1, 10)\) may not be uniformly continuous.
(b). Let \( f(x) \) be a function defined in \([a, b]\). If \( f(x) \) is continuous in \((a, b)\) and \( f(x) \) is differentiable at \( a \) and \( b \), prove that \( f(x) \) is uniformly continuous in \([a, b]\).


Derivatives for functions (Chapter 6).
Basic algebraic properties and Chain rule:

Exercise 7: (a). If \( f(x) \) is differentiable at \( x = c \in I \), where \( I \) is an interval, prove that \( f(x)^2 \) is differentiable at \( x = c \in I \). Show that \( f(x)^{1/3} \) might not be differentiable at \( x = c \in I \).
(b). The proof of Chain rule.

Mean value theorem: Various forms

Exercise 8: Assume that \( f(x) \) and \( g(x) \) are two continuous function in \([a, b]\). If \( f(x) \) and \( g(x) \) are differentiable in \((a, b)\), and \( g'(x) \neq 0 \) in \((a, b)\), show that there is a \( c \in (a, b) \) such that

\[
\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}.
\]

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!
Comments and question to: mzhu@math.ou.edu
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