Review for Midterm 1

Extra office hour: Th 2:00-3:30pm

On Preliminaries (Chapter 1).

Set and functions: De Morgan Law, operations for sets, functions.

Exercise 1: Find $\bigcap_{i=1}^{\infty} A_i$ when
(a). $A_i = [0, \frac{1}{n}]$, (b). $A_i = [0, \frac{1}{n})$, (c). $A_i = (0, \frac{1}{n}]$, (d). $A_i = (0, \frac{1}{n})$

Math induction: standard and strong induction, why it works.

(b). Prove that $5^{2n} - 1$ is divisible for all natural number $n$.

Finite and infinite sets: Definitions. existence of uncountable sets (relation to Nested Interval Property) and Cantor’s theorem.

Exercise 3: Is there any bijection between the set of all positive rational number and the set of natural number? If yes, give an example.

On Real numbers (Chapter 2).

Basic algebraic properties: Algebraic operations and Absolute values.

Exercise 4: Let $f(x)$ be a function defined in an interval $I$. Show that there are an odd function $g(x)$ and an even function $h(x)$ in $I$ such that

$$f(x) = g(x) + h(x).$$

Completeness and Supremum (infimum): Completeness axiom, Supremum and Infimum, Archimedean principle, density, intervals.

Exercise 5:
(a). Let $A$ be a set of some positive numbers. Prove that
\[ \inf \frac{1}{A} = \frac{1}{\sup A}. \]
where $\frac{1}{A} = \{\frac{1}{a} : a \in A\}$.

(b). If $\lim x_n = \sup\{x_n : n \in N\}$, is $x_{n+m}$ an increasing sequence for large natural number $m$?

**On Sequences (Chapter 3).** limit: Definition, operation, convergence, divergence.

**Exercise 6**: Using definition to prove
\[ \lim \sqrt{1 + \frac{1}{n}} = 1. \]

Monotone and subsequences Convergence for monotonic and bounded sequences, subsequences

**Exercise 7**: (a). Let $a$ be a positive number. If $x_1 \in (0, 1/a)$ and
\[ x_{n+1} = x_n(2 - ax_n), \]
show that $\lim x_n$ exists. What is it?

(b). (Hardest one) If $x_1 = 1$ and
\[ x_{n+1} = \frac{1}{1 + x_n}, \]
prove $\lim x_n = (\sqrt{5} - 1)/2$. (That is: prove the existence of limit first, then compute the limit.)

**WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!**

Comments and question to: mzhu@math.ou.edu