Review for the course

Final Exam: 12-13-2005, 1:30pm-3:30pm.

Extra office hour: Monday 10:30am-2:30pm

Complex numbers and functions, Chapter 2.
Complex numbers: Definitions, two forms, algebraic operations.

Exercise 1: Find the real and imaginary parts and modules of following numbers:
(a). \(z = (1 + i)^{-2i}\); (b). \(z = e^{2+4i}\).

Fourier series, Chapter 7
Harmonic functions, periodic functions: Period, amplitude, ”orthogonal” of two harmonic functions.

Exercise 2: Computer

\[
\int_{-\pi}^{\pi} e^{2xi} \cdot e^{-3xi} dx = ?
\]

Fourier series: How to find Fourier coefficients. Parseval’s theorem and applications.

Exercise 3:
(a). Find Fourier series for

\[
f(x) = \begin{cases} 
-1, & -l \leq x < 0, \\
1, & 0 \leq x < l.
\end{cases}
\]

(b). Using above Fourier series and Parseval’s theorem to computer

\[1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\]

First order ODEs
Definition: We start with some terminologies: Differential equations, solutions, general solutions, particular solutions, etc.

**Exercise 4:** Check that \( y = ce^{-ax} \) is a general solution to
\[
y'' - \alpha^2y = 0.
\]

How to solve a differential equation (1st order): 7 types:

**Exercise 5:** Solve
\[
y^2 \frac{dy}{dx} + 2xy^3 = x.
\]

**High order linear ODEs**

Definition: We start with some terminologies: Linear differential equations, superposition, general solutions, homogeneous and nonhomogeneous equations, particular solutions, etc.

**Exercise 6:** If \( y_1 \) and \( y_2 \) solve equation
\[
y'' + e^x y' + y^2 \sin x = 0,
\]
is \( y_1 + y_2 \) also a solution? If \( y_1 + y_2 \) is still a solution, show that
\[
y_1(x)y_2(x) \sin x = 0.
\]

How to solve a homogeneous equation of constant coefficients:
\[
y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1y' + a_0y = 0.
\]

Three types.

**Exercise 7:** If the characteristic equation of a ODE is
(a).
\[
(r - 1)^3(r^2 - 4x + 5) = 0,
\]
(b).
\[
(r - 1)^3(r^2 - 2x + 2)^2 = 0,
\]
find the general solutions.

How to solve a nonhomogeneous equation of constant coefficients:
\[
y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1y' + a_0y = f(x).
\]
Undetermined coefficient method: Three cases.

**Exercise 8**: Find the general solution to

\[ y'' + 4y' + 5 = x + e^{2x} \sin x. \]

solve an equation of constant coefficients by **Laplace transform**:

**Exercise 8**: Using Laplace transform to find the solution to

\[ y'' + 4y' + 5 = x + e^{2x} \sin x, \quad y'(0) = 0, \quad y''(0) = 1. \]

**Exercise 9**: If

\[ Y = \frac{p + 4}{(p + 4)^2 + 3}, \]

find \( y(t) \) so that \( L\{y(t)\} = Y. \)

**Eigenvalue problems for boundary value problems**: the set-up and its applications to solving PDEs.

Typical example:

**Exercise 10**: Solve the eigenvalue problems:

\[ y'' = \lambda y, \quad y'(0) = y' (\pi) = 0. \]

Here is a fun one:

**Exercise 11**: Solve the eigenvalue problems:

\[ y' = \lambda y, \quad y(0) = 1, \quad y(1) = e. \]

**Power series to solve ODEs**: how to find the recursive relation.

**Exercise 12** Using power series to solve

\[ y'' = y. \]

(a). Find the recursive relation.

(b). Find the power series solution.

**PDEs**:

Laplace equation, heat equation and wave equation. Two steps. Step 1: solving homogeneous boundary problem with the help of the solution to eigenvalue problem. Step 2: Solving initial value of whole boundary problem.
Exercise 13 Find three linearly independent solutions to

\[ u_x - u_y = 0. \]

Exercise 14 Find three linearly independent solutions to

\[ u_x(x, y) - u_y(x, y) = 0, \text{ and } u(0, y) = u(\pi, y) = 0. \]

Formulas will be provided: Fourier series formula, Laplace transform formulas.