Partial derivative
We start to learn functions of several variables.

Definition:
How to find Domain, what is Level curves (and Level curves, etc.

Limit and Continuity:
When a rational polynomial has NO limit: Find two difference paths.
How to find limit? Definition is TOO HARD. But if the function is continuous at the point, it is easy!

Try these examples
Exercise 1: If the limit exists, find it. If it does not exist, give the reason.
1).
\[ \lim_{(x,y) \to (0,0)} \frac{x^{10} + y \ln(x^2 + 1)}{x^3 + y^6 + 1}. \]

2)
\[ \lim_{(x,y) \to (0,0)} \ln(x^2 + y^3 + 2). \]

3)
\[ \lim_{(x,y) \to (0,0)} \frac{x^4 y}{x^8 + y^2}. \]

Partial derivative:
How to compute the partial derivatives (include how to check a function solving a partial differential equations): All those old tricks from previous Calculus courses are still useful. Key part: Chain rule.
Exercise 2: Check \( u(x,t) = f(x + at) \) solves the following equation:

\[
u_{tt} - a^2 u_{xx} = 0.
\]

Applications of derivatives: Tangent plane and normal line. The general way to find the formula using level surface: Where the normal direction

\[
(F_x(x_0, y_0, z_0), F_y(x_0, y_0, z_0), F_z(x_0, y_0, z_0))
\]

come from?

Exercise 3: Assume that \( g(x, y) = f(x^2 + \sin y) \) where \( f(t) \) is a differentiable function of \( t \). If \( f'(0) = 2 \),
(a) Find \( \partial g / \partial y(0,0) = ? \).
(b) Find the tangent plane of \( g(x, y) \) at point \( (0,0) \).

Maximum and Minimum

How to find Critical Points: Solve the system of equations.

How to distinguish the local MAX and MIN from critical points: compute the determinant and \( f_{xx} \).

How to find GLOBAL extreme value: 1). fine extreme value insider the region, 2). find with constraint using Lagrange multipliers.

Exercise 4. Typical example: find the maximum and minimum value of \( f(x, y) = xy \) in the closed disk: \( \{(x, y) : x^2 + y^2 \leq 1\} \).
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