Hints for the Review

**Exercise 1.** For what values of $b$ is the vector $(1, b, -2)$ is parallel to the plane $2x + 5y - z = d$?

**Hint:**

$$(1, b, -2) \cdot (2, 5, -1) = 0,$$

thus $b = -4/5$.

**Exercise 2.** (a) Find $i \times (j \times k) = ?$

**Hint:** $= i \times i = 0$.

$b \times (i \times k) = ?$

**Hint:** $= i \times (-j) = -k$.

(b) Assume $a \neq 0$. If $a \cdot b = a \cdot c$, is $b = c$?

**Hint:** NO. Please find example by yourself.

Line and plane: Equations, geometric meaning.

**Exercise 3:** (a) Do the following two line intersect each other? If yes, find the intersection point: $r = (1, 2, 3) + t(2, 3, 1); \ r = (2, 2, 1) + t(1, 3, 1)$.

**Hint:**

$$(1, 2, 3) + t(2, 3, 1) = (2, 2, 1) + s(1, 3, 1),$$

thus

$$(1, 0, -2) = (2t - s), 3t - 3s, t - s).$$

It has no solution.

(b) For what values of $a$ do the following two line intersect each other $r = (1, 2, 3) + t(2, 3, 1); \ r = (2, 2, 1) + t(1, 3, a)$?

**Hint:** Check the solution for part (a).
Exercise 4: (a). Show that the distance between the given parallel planes \( ax + by + cz = d_1 \) and \( ax + by + cz = d_2 \) is

\[
\text{distance} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.
\]

Hint: Assume that \( P(x_1, y_1, z_1) \) is a point on the first plane. Thus \( ax_1 + by_1 + cz_1 = d_1 \). Since the distance from point \( P \) to the second plane is given by

\[
\text{distance} = \frac{|ax_1 + by_1 + cz_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}},
\]

thus

\[
\text{distance} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.
\]

(b). Find the equations of the planes which is parallel to \( x + 2y + 3z = 1 \) and have distance 3 to this plane.

Hint: It is:

\[
x + 2y + 3z = 1 + 3\sqrt{14}
\]
or

\[
x + 2y + 3z = 1 - 3\sqrt{14}.
\]

WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!

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