Review for Midterm 3

Extra office hour: Monday 9:30-11:00am

On Parametric equations and Polar coordinates (Chapter 11).

Area and Length: Intersection point(s), Area and Length using polar coordinates.

Exercise 1: (a). Find ALL intersection points: \( r = 2 \) and \( r = 2 \cos 2\theta \).
(b). Find the area of the region that lies inside both of the circles \( r = 2 \sin \theta \) and \( r = \sin \theta + \cos \theta \).

On (Vectors and geometry of Spaces Chapter 13).

Vectors: Algebraic operation, vector products (Dot and cross, and scalar triple). Geometric meanings (addition, subtraction, dot product, cross product and mixed product).

Exercise 2. For what values of \( b \) is the vector \((1, b, -2)\) is parallel to the plane \(2x + 5y - z = d?\)

Exercise 3. (a) Find
\[
\mathbf{i} \times (\mathbf{j} \times \mathbf{k}) = ?
\]
\[
\mathbf{i} \times (\mathbf{i} \times \mathbf{k}) = ?
\]

(b) Can you prove that
\[
\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}?
\]

(c) Assume \( \mathbf{a} \neq \mathbf{0} \). If \( \mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \), is \( \mathbf{b} = \mathbf{c} \)?

Line and plane: Equations, geometric meaning.

Exercise 3: (a) Do the following two line intersect each other? If yes, find the intersection point: \( \mathbf{r} = (1, 2, 3) + t(2, 3, 1) \); \( \mathbf{r} = (2, 2, 1) + t(1, 3, 1) \).
(a) For what values of $a$ do the following two line intersect each other $\mathbf{r} = (1, 2, 3) + t(2, 3, 1); \quad \mathbf{r} = (2, 2, 1) + t(1, 3, a)$?

**Exercise 4**: (a). Show that the distance between the given parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ is

$$\text{distance} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

(b). Find the equations of the planes which is parallel to $x + 2y + 3z = 1$ and have distance 3 to this plane.

**WARNING: YOU ARE RESPONSIBLE FOR CHECKING OUT MY TYPOS!**

Comments and question to: mzhu@math.ou.edu