3. Vertical format for subtraction

Negative coefficient notations:

$2\hat{2} = 2 \times 10 - 2 \times 1 = 20 - 2 = 18;$ .... ⑩

$4\hat{3}1 = 4 \times 10^2 - 3 \times 10 + 1 = 400 - 30 + 1 = 371$ .... ⑪

$3\hat{4}\hat{7} = 3 \times 10^2 - 4 \times 10 - 7 = 300 - 40 - 7 = 26\hat{7} = 253$ .... ⑫

$\hat{5}\hat{8} = -5 \times 10 - 8 = -58 = \hat{6}2$ .... ⑬

Explanation:

In the above examples, we introduce **negative coefficient notations**:

- ⑩ represents a two-digit number where the ones place has a negative coefficient (-2), same as the traditional decimal number 18.
- ⑪ represents a three-digit number where the tens place has a negative coefficient (-3), same as 371 in the traditional decimal system.
- ⑫ represents another three-digit number with negative coefficients in both the tens place (-4) and ones place (-7), same as 253 in the traditional decimal system.
- ⑬ represents a two-digit number with negative coefficients in both the tens place (-5) and ones place (-8), which is the same as -58. This number can also be written using the new notation as $\hat{6}2$.

Here are three examples.

Example 3.1. Using vertical form to calculate 32-14.

Solution:

		Tens place	Ones place		
		3	2		
	-	1	4		
Ones place			-2	……..	⑭
Tens place	+	2		……..	⑮
Final answer		2	$\hat{2}$	(= 18) ……	⑯

Explanation:

1. **Step ⑭:** Subtract the ones place digits: 2−4=−2. The result (**-**2) is placed in the ones column.
2. **Step ⑮:** Subtract the tens place digits: 3−1=2. The result (2) is placed in the tens column.
3. **Step ⑯:** Combining the results from Steps ⑭ and ⑮, we obtain $2\hat{2}$, which is the same as 18 in the traditional decimal system.

Example 3.2. Using vertical form to calculate 434-172.

Solution:

		Hundreds	Tens	Ones		
		4	**3**	**4**		
	-	**1**	**7**	**2**		
Ones place				**2**	**.........**	⑰
Tens place			$\hat{4}$		**.........**	⑱
Hundreds	**+**	**3**			**.........**	⑲
Final answer		**3**	$\hat{4}$	**2**	**(= 262)**........	⑳

Explanation:

1. **Step ⑰:** Subtract the ones place digits: 4−2=2. The result (2) is placed in the ones column.
2. **Step ⑱:** Subtract the tens place digits: 3−7=−4. The result ($\hat{4}$) is placed in the tens column.
3. **Step ⑲:** Subtract the hundreds place digits: 4−1=3. The result (3) is placed in the hundreds column.
4. **Step ⑳:** Combining the results from Steps ⑰, ⑱, and ⑲, we obtain $3\hat{4}2$, which is the same as 262 in the traditional decimal system.

Example 3.3. Using vertical form to calculate $45\hat{1}$-183.

Solution:

		Hundreds	Tens	Ones	
		4	**5**	$\hat{1}$	
	-	**1**	**8**	**3**	
Ones place				$\hat{4}$	**……..** Ⓐ
Tens place			$\hat{3}$		**……..** Ⓑ
Hundreds	**+**	**3**			**………** ©
Final answer		**3**	$\hat{3}$	$\hat{4}$	**(= 266)…..** Ⓓ

Explanation:

1. **Step Ⓐ:** Subtract the ones place digits: $\hat{1}$- 3 = $\hat{4}$. The result ($\hat{4}$) is placed in the ones column.
2. **Step Ⓑ:** Subtract the tens place digits: 5−8= $\hat{3}$. The result ($\hat{3}$) is placed in the tens column.
3. **Step ©:** Subtract the hundreds place digits: 4−1=3. The result (3) is placed in the hundreds column.
4. **Step Ⓓ:** Combining the results from Steps Ⓐ, Ⓑ, and ©, we obtain $3\hat{3}\hat{4}$, which is the same as 266 in the traditional decimal system.

Example 4.3 (With Negative Coefficients and Carrying Numbers): Using vertical form to calculate $3\hat{6} \times 26$.

Solution:

		Hundreds	Tens	Ones	
			3	$\hat{6}$	
	$\times$		**2**	**6**	
Ones-Ones			$\hat{3}$	$\hat{6}$	………. Ⓢ
Ones- Tens		**1**	**8**		………. Ⓣ
Tens-Ones		$\hat{1}$	$\hat{2}$		………. Ⓤ
Tens-Tens	$+$	**6**			………. Ⓥ
Final answer		**6**	**3**	$\hat{6}$	**(= 624) ….** Ⓦ

Explanation:

1. **Step Ⓢ:** Multiply the ones place digits: $\hat{6}\times6=\hat{3}\hat{6}$. The result ($\hat{3}\hat{6}$) is placed in the horizontal ones-ones place.
2. **Step Ⓣ:** Multiply the ones digit of the multiplier by the tens digit of the multiplicand: $6\times2=18$. The result (18) is placed in the horizontal ones-tens place.
3. **Step Ⓤ:** Multiply the tens digit of the multiplier by the ones digit of the multiplicand: $2\times\hat{6}=\hat{1}\hat{2}$. The result ($\hat{1}\hat{2}$) is placed in the horizontal tens-ones place.
4. **Step Ⓥ:** Multiply the tens place digits: $2\times3=6$. The result (6) is placed in the vertical hundreds place and horizontal tens-tens place.
5. **Final Step Ⓦ:** Adding all the partial results together gives the final answer ($63\hat{6}$), which is the same as traditional decimal number 624 in the parenthesis.

Example 4.4. Using vertical form to calculate $4\hat{2}1 \times 21$.

Solution:

		4	$\hat{\mathbf{2}}$	**1**
$\times$			**2**	**1**
Ones-ones				**1**
Ones-tens			$\hat{\mathbf{2}}$	
Ones-hundreds		**4**		
Tens -ones			**2**	
Tens-tens		$\hat{\mathbf{4}}$		
Tens-hundreds +	8			
Final answer	**8**	**0**	**0**	**1** .

9. Application for factorization

Example 9.1. 299 cf (2x+3)(x+3)

Meaning: The first term= $2 \times 1 = 2$, the cross term=$2 \times 3 + 1 \times 3 = 9$, the last term= $3 \times 3 = 9$.

$$\begin{array}{ccc} 2 & & 3 \\ & \times & \\ 1 & & 3 \end{array}$$

So, we can factorize $299 = 23 \times 13$; Similarly: $2x^2 + 9x + 9 = (2x + 3)(x + 3)$.

Example 9.2: $\mathbf{25\hat{3}\ (= 247)}$ **cf (x+3)(2x-1) ;**

Meaning: The first term= $1 \times 2 = 2$, the cross term=$1 \times (-1) + 2 \times 3 = 5$, the last term= $3 \times (-1) = -3 = \hat{3}$.

$$\begin{array}{ccc} 1 & & 3 \\ & \times & \\ 2 & & \hat{1} \end{array}$$

So, we can factorize $25\hat{3} = 13 \times 2\hat{1}$ (=13× 19). It is not easy to observe 247=13× 19! Similarly: $2x^2 + 5x - 3 = (x + 3)(2x - 1)$.