

### 3. Vertical format for subtraction

**Negative coefficient notations:**

$$2\hat{2} = 2 \times 10 - 2 \times 1 = 20 - 2 = 18; \quad \text{..... } \textcircled{10}$$

$$4\hat{3}1 = 4 \times 10^2 - 3 \times 10 + 1 = 400 - 30 + 1 = 371 \quad \text{..... } \textcircled{11}$$

$$3\hat{4}\hat{7} = 3 \times 10^2 - 4 \times 10 - 7 = 300 - 40 - 7 = 26\hat{7} = 253 \quad \text{..... } \textcircled{12}$$

$$\hat{5}\hat{8} = -5 \times 10 - 8 = -58 = \hat{6}2 \quad \text{..... } \textcircled{13}$$

**Explanation:**

In the above examples, we introduce **negative coefficient notations**:

- $\textcircled{10}$  represents a two-digit number where the ones place has a negative coefficient (-2), same as the traditional decimal number 18.
- $\textcircled{11}$  represents a three-digit number where the tens place has a negative coefficient (-3), same as 371 in the traditional decimal system.
- $\textcircled{12}$  represents another three-digit number with negative coefficients in both the tens place (-4) and ones place (-7), same as 253 in the traditional decimal system.
- $\textcircled{13}$  represents a two-digit number with negative coefficients in both the tens place (-5) and ones place (-8), which is the same as -58. This number can also be written using the new notation as  $\hat{6}2$ .

Here are three examples.

**Example 3.1.** Using vertical form to calculate  $32-14$ .

**Solution:**

		Tens place	Ones place	
		3	2	
	-	1	4	
Ones place			-2	..... ⑭
Tens place	+	2		..... ⑮
Final answer		2	$\hat{2}$	( = 18 ) ..... ⑯

**Explanation:**

1. **Step ⑭:** Subtract the ones place digits:  $2-4=-2$ . The result (-2) is placed in the ones column.
2. **Step ⑮:** Subtract the tens place digits:  $3-1=2$ . The result (2) is placed in the tens column.
3. **Step ⑯:** Combining the results from Steps ⑭ and ⑮, we obtain  $2\hat{2}$  , which is the same as 18 in the traditional decimal system.

**Example 3.2.** Using vertical form to calculate  $434-172$ .

**Solution:**

	Hundreds	Tens	Ones	
	4	3	4	
-	1	7	2	
Ones place			2	..... ⑰
Tens place		$\hat{4}$		..... ⑱
Hundreds	+	3		..... ⑲
Final answer	3	$\hat{4}$	2	( = 262 ) ..... ㉔

**Explanation:**

1. **Step ⑰:** Subtract the ones place digits:  $4-2=2$ . The result (2) is placed in the ones column.
2. **Step ⑱:** Subtract the tens place digits:  $3-7=-4$ . The result ( $\hat{4}$ ) is placed in the tens column.
3. **Step ⑲:** Subtract the hundreds place digits:  $4-1=3$ . The result (3) is placed in the hundreds column.
4. **Step ㉔:** Combining the results from Steps ⑰, ⑱, and ⑲, we obtain  $3\hat{4}2$ , which is the same as 262 in the traditional decimal system.

**Example 3.3.** Using vertical form to calculate  $45\hat{1}-183$ .

**Solution:**

	Hundreds	Tens	Ones	
	4	5	$\hat{1}$	
-	1	8	3	
<hr style="border: 1px solid black;"/>				
Ones place			$\hat{4}$	..... ①
Tens place		$\hat{3}$		..... ②
Hundreds	+	3		..... ③
<hr style="border: 1px solid black;"/>				
Final answer	3	$\hat{3}$	$\hat{4}$	( = 266)..... ④

**Explanation:**

1. **Step ①:** Subtract the ones place digits:  $\hat{1}-3=\hat{4}$ . The result ( $\hat{4}$ ) is placed in the ones column.
2. **Step ②:** Subtract the tens place digits:  $5-8=\hat{3}$ . The result ( $\hat{3}$ ) is placed in the tens column.
3. **Step ③:** Subtract the hundreds place digits:  $4-1=3$ . The result (3) is placed in the hundreds column.
4. **Step ④:** Combining the results from Steps ①, ②, and ③, we obtain  $3\hat{3}\hat{4}$ , which is the same as 266 in the traditional decimal system.

**Example 4.3 (With Negative Coefficients and Carrying Numbers):** Using vertical form to calculate  $3\hat{6} \times 26$ .

**Solution:**

	Hundreds	Tens	Ones	
		3	$\hat{6}$	
		2	6	
<hr/>				
Ones-Ones		$\hat{3}$	$\hat{6}$	..... ⑤
Ones- Tens	1	8		..... ①
Tens-Ones	$\hat{1}$	$\hat{2}$		..... ②
Tens-Tens	+	6		..... ④
<hr/>				
Final answer	6	3	$\hat{6}$	( = 624 ) .... ⑥

**Explanation:**

1. **Step ⑤:** Multiply the ones place digits:  $\hat{6} \times 6 = 3\hat{6}$ . The result ( $\hat{3}\hat{6}$ ) is placed in the horizontal ones-ones place.
2. **Step ①:** Multiply the ones digit of the multiplier by the tens digit of the multiplicand:  $6 \times 2 = 18$ . The result (18) is placed in the horizontal ones-tens place.
3. **Step ②:** Multiply the tens digit of the multiplier by the ones digit of the multiplicand:  $2 \times \hat{6} = \hat{1}\hat{2}$ . The result ( $\hat{1}\hat{2}$ ) is placed in the horizontal tens-ones place.
4. **Step ④:** Multiply the tens place digits:  $2 \times 3 = 6$ . The result (6) is placed in the vertical hundreds place and horizontal tens-tens place.
5. **Final Step ⑥:** Adding all the partial results together gives the final answer ( $63\hat{6}$ ), which is the same as traditional decimal number 624 in the parenthesis.

**Example 4.4.** Using vertical form to calculate  $4\hat{2}1 \times 21$ .

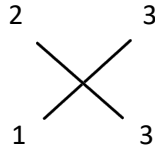
**Solution:**

$$\begin{array}{r}
 \begin{array}{r}
 4 \quad \hat{2} \quad 1 \\
 \times \quad 2 \quad 1 \\
 \hline
 \end{array} \\
 \begin{array}{l}
 \text{Ones-ones} \qquad \qquad \qquad 1 \\
 \text{Ones-tens} \qquad \qquad \qquad \hat{2} \\
 \text{Ones-hundreds} \qquad \qquad 4 \\
 \text{Tens -ones} \qquad \qquad \qquad 2 \\
 \text{Tens-tens} \qquad \qquad \qquad \hat{4} \\
 \text{Tens-hundreds} \quad + \quad 8 \\
 \hline
 \end{array} \\
 \begin{array}{r}
 \text{Final answer} \qquad 8 \qquad 0 \qquad 0 \qquad 1 \quad .
 \end{array}
 \end{array}$$

## 9. Application for factorization

### Example 9.1. 299 cf $(2x+3)(x+3)$

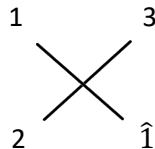
Meaning: The first term=  $2 \times 1 = 2$ , the cross term= $2 \times 3 + 1 \times 3 = 9$ , the last term=  $3 \times 3 = 9$ .



So, we can factorize  $299 = 23 \times 13$ ; Similarly:  $2x^2 + 9x + 9 = (2x + 3)(x + 3)$ .

### Example 9.2: $25\hat{3} (= 247)$ cf $(x+3)(2x-1)$ ;

Meaning: The first term=  $1 \times 2 = 2$ , the cross term= $1 \times (-1) + 2 \times 3 = 5$ , the last term=  $3 \times (-1) = -3 = \hat{3}$ .



So, we can factorize  $25\hat{3} = 13 \times 2\hat{1} (=13 \times 19)$ . It is not easy to observe  $247=13 \times 19$ ! Similarly:  $2x^2 + 5x - 3 = (x + 3)(2x - 1)$ .