Homework 5 Solutions

(1) $F(x)=(1+x+x^2)^{99}$

$f(x)=x^{99}$ $\quad g(x)=1+x+x^2$

$f'(x)=99x^{98}$ $\quad g'(x)=1+2x$

$$\boxed{F'(x)=99(1+x+x^2)^{98}(1+2x)}$$

(2) • $f(x)=\dfrac{1}{\sqrt[3]{x^2-1}}$

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$=\dfrac{1}{(x^2-1)^{1/3}}$

$=(x^2-1)^{-1/3}$

$g(x)=x^{-1/3}$ $\quad h(x)=x^2-1$

$g'(x)=-\frac{1}{3}x^{-4/3}$ $\quad h'(x)=2x$

$$\boxed{f'(x)=-\frac{1}{3}(x^2-1)^{-4/3}\cdot 2x}$$

(3) $y=\left(x+\dfrac{1}{x}\right)^5$

$f(x)=x^5$ $\quad g(x)=x+\frac{1}{x}$

$f'(x)=5x^4$ $\quad g(x)=x+x^{-1}$

$g'(x)=1-x^{-2}$

$$\boxed{y'=5\left(x+\frac{1}{x}\right)^4(1-x^{-2})}$$

(4) $y=\sin(t+\cos(\sqrt{t}))$

$f(x)=\sin(x)$ $\quad g(t)=t+\cos(\sqrt{t})$

$f'(x)=\cos(x)$ $\quad g'(t)=1+(\cos(\sqrt{t}))'$

$F(x) = \cos(\sqrt{t})$

$f(x) = \cos(x) \quad g(t) = \sqrt{t}$

$f'(x) = -\sin(x) \quad g(t) = t^{1/2}$

$g'(t) = \frac{1}{2} t^{-1/2}$

$(\cos(\sqrt{t}))' = -\sin(\sqrt{t})\left(\frac{1}{2} t^{-1/2}\right)$

So,

$g'(t) = 1 - \sin(\sqrt{t})\left(\frac{1}{2} t^{-1/2}\right)$

$$\boxed{y' = \cos(t + \cos(\sqrt{t}))\left(1 - \sin(\sqrt{t})\left(\frac{1}{2} t^{-1/2}\right)\right)}$$

5 $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$f(x) = \sqrt{x} \quad g(x) = x + \sqrt{x + \sqrt{x}}$

$f(x) = x^{1/2} \quad g'(x) = 1 + \left(\sqrt{x + \sqrt{x}}\right)'$

$f'(x) = \frac{1}{2} x^{-1/2}$

$F(x) = \sqrt{x + \sqrt{x}}$

$f(x) = \sqrt{x} \quad g(x) = x + \sqrt{x}$

$f(x) = x^{1/2} \quad g(x) = x + x^{1/2}$

$f'(x) = \frac{1}{2} x^{-1/2} \quad g'(x) = 1 + \frac{1}{2} x^{-1/2}$

$F'(x) = \frac{1}{2}(x + \sqrt{x})^{-1/2}\left(1 + \frac{1}{2} x^{-1/2}\right)$

So,

$g'(x) = 1 + \frac{1}{2}(x + \sqrt{x})^{-1/2}\left(1 + \frac{1}{2} x^{-1/2}\right)$

$$\boxed{y' = \frac{1}{2}\left(x + \sqrt{x + \sqrt{x}}\right)^{-1/2}\left(1 + \frac{1}{2}(x + \sqrt{x})^{-1/2}\left(1 + \frac{1}{2} x^{-1/2}\right)\right)}$$

16 $2x^2+x+xy=1$

(a) $4x+1+y+x\dfrac{dy}{dx}=0$

$x\dfrac{dy}{dx}=-4x-1-y$

$$\boxed{\frac{dy}{dx}=\frac{-4x-1-y}{x}}$$

(b) $xy=1-2x^2-x$

$y=\dfrac{1}{x}-2x-1$

$$\boxed{y'=-\frac{1}{x^2}-2}$$

(c) $$\frac{dy}{dx}=\frac{-4x-1-\left(\frac{1}{x}-2x-1\right)}{x}$$

$$=\frac{-4x-\cancel{1}-\frac{1}{x}+2x+\cancel{1}}{x}$$

$$=\frac{-2x-\frac{1}{x}}{x}$$

$$\boxed{\frac{dy}{dx}=-2-\frac{1^{2}}{x}}$$

7 (a) $\frac{2}{x} - \frac{1}{y} = 4$

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$$2x^{-1} - y^{-1} = 4$$

$$-2x^{-2} + y^{-2}y' = 0$$

$$y^{-2}y' = 2x^{-2}$$

$$y' = \frac{2x^{-2}}{y^{-2}}$$

$$\boxed{y' = \frac{2y^2}{x^2}}$$

(b) $-\frac{1}{y} = 4 - \frac{2}{x}$

$$-1 = y\left(4 - \frac{2}{x}\right)$$

$$\frac{-1}{4 - \frac{2}{x}} = y$$

$$y = \frac{-1}{\frac{4x-2}{x}}$$

$$y = \frac{-x}{4x-2}$$

$$y' = \frac{-(4x-2) - 4(-x)}{(4x-2)^2}$$

$$= \frac{-\cancel{4x} + 2 + \cancel{4x}}{(4x-2)^2}$$

$$\boxed{y' = \frac{2}{(4x-2)^2}}$$

(c) $y' = \frac{2\left(\frac{-x}{4x-2}\right)^2}{x^2}$

$$y' = \frac{2\left(\frac{x^2}{(4x-2)^2}\right)}{x^2}$$

$$y' = \frac{2x^2}{(4x-2)^2 x^2}$$

$$\boxed{y' = \frac{2}{(4x-2)^2}}$$

⑧ $\cos(xy) = 1+\sin(y)$

$-\sin(xy)(y+xy') = \cos(y)y'$

$-\sin(xy)y + -\sin(xy)xy' = \cos(y)y'$

$-\sin(xy)y = \cos(y)y' + \sin(xy)xy'$

$-\sin(xy)y = y'(\cos(y)+\sin(xy)x)$

$$\boxed{y' = \frac{-\sin(xy)y}{\cos(y)+\sin(xy)x}}$$

⑨ $xy = \sqrt{x^2+y^2}$

$xy'+y = \frac{1}{2}(x^2+y^2)^{-1/2}(2x+2yy')$

$xy'+y = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x + \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2yy'$

$xy' - \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cancel{2}yy' = \frac{1}{\cancel{2}}(x^2+y^2)^{-1/2}\cdot\cancel{2}x - y$

$(x-(x^2+y^2)^{-1/2}y)y' = (x^2+y^2)^{-1/2}x - y$

$$\boxed{y' = \frac{(x^2+y^2)^{-1/2}x - y}{x-(x^2+y^2)^{-1/2}y}}$$

10 $\tan(x-y) = \dfrac{y}{1+x^2}$

$$\sec^2(x-y)(1-y') = \frac{y'(1+x^2)-y(2x)}{(1+x^2)^2}$$

$$\sec^2(x-y) - \sec^2(x-y)y' = \frac{y'+y'x^2-2xy}{(1+x^2)^2}$$

$$(1+x^2)^2(\sec^2(x-y)-\sec^2(x-y)y') = y'+y'x^2-2xy$$

$$(1+x^2)^2\sec^2(x-y)-(1+x^2)^2\sec^2(x-y)y' = y'+y'x^2-2xy$$

$$(1+x^2)^2\sec^2(x-y)+2xy = y'+y'x^2+(1+x^2)^2\sec^2(x-y)y'$$

$$(1+x^2)^2\sec^2(x-y)+2xy = y'(1+x^2+(1+x^2)^2\sec^2(x-y))$$

$$\boxed{y' = \frac{(1+x^2)^2\sec^2(x-y)+2xy}{1+x^2+(1+x^2)^2\sec^2(x-y)}}$$

11 $y\sec(x) = x\tan(y)$

$$\sec(x)+y\sec(x)\tan(x)\frac{dx}{dy} = \tan(y)\frac{dx}{dy}+x\sec^2(y)$$

$$y\sec(x)\tan(x)\frac{dx}{dy}-\tan(y)\frac{dx}{dy} = x\sec^2(y)-\sec(x)$$

$$\frac{dx}{dy}(y\sec(x)\tan(x)-\tan(y)) = x\sec^2(y)-\sec(x)$$

$$\boxed{\frac{dx}{dy} = \frac{x\sec^2(y)-\sec(x)}{y\sec(x)\tan(x)-\tan(y)}}$$

12 $x^2+xy+y^2=3$

$$2x + xy' + y + 2yy' = 0$$

$$xy' + 2yy' = -2x - y$$

$$(x+2y)y' = -2x - y$$

$$y' = \frac{-2x-y}{x+2y}$$

$$y'' = \frac{(-2+y')(x+2y) - (-2x-y)(1+2y')}{(x+2y)^2}$$

$$y'' = \frac{-2x - 4y + xy' - (-2x - 4xy' - y - 2yy')}{(x+2y)^2}$$

$$y'' = \frac{\cancel{-2x} - 4y + xy' + \cancel{2x} + 4xy' + y + 2yy'}{(x+2y)^2}$$

$$y'' = \frac{-3y + 5xy' + 2yy'}{(x+2y)^2}$$

$$y'' = \frac{-3y + y'(5x+2y)}{(x+2y)^2}$$

$$y'' = \frac{-3y + \left(\frac{-2x-y}{x+2y}\right)(5x+2y)}{(x+2y)^2}$$

$$y'' = \frac{-3y + \frac{(-10x^2 - 4xy - 5xy - 2y^2)}{(x+2y)}}{(x+2y)^2}$$

$$y'' = \frac{-3y + \dfrac{(-10x^2 - 4xy - 5xy - 2y^2)}{(x+2y)}}{(x+2y)^2}$$

$$y'' = \frac{-3y(x+2y) - 10x^2 - 9xy - 2y^2}{(x+2y)^3}$$

$$y'' = \frac{-3xy - 6y^2 - 10x^2 - 9xy - 2y^2}{(x+2y)^3}$$

$$y'' = \frac{-12xy - 8y^2 - 10x^2}{(x+2y)^3}$$

$$y'' = \frac{-8y^2 - 8xy - 8x^2 - 4xy - 2x^2}{(x+2y)^3}$$

$$y'' = \frac{-8(y^2 + xy + x^2) - 4xy - 2x^2}{(x+2y)^3}$$

$$y'' = \frac{-8(3) - 4xy - 2x^2}{(x+2y)^3}$$

$$\boxed{y'' = \frac{-24 - 4xy - 2x^2}{(x+2y)^3}}$$

(13) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} + \frac{2y \cdot y'}{b^2} = 0$$

$$\frac{2y \cdot y'}{b^2} = \frac{-2x}{a^2}$$

$$y' = -\frac{\cancel{2}x}{a^2} \cdot \frac{b^2}{\cancel{2}y}$$

$$y' = \frac{-x \cdot b^2}{y \cdot a^2}$$

At (x_0, y_0): $m = \frac{-x_0 b^2}{y_0 a^2}$

$$(y - y_0) = \frac{-x_0 b^2}{y_0 a^2}(x - x_0)$$

$$(y - y_0)(y_0 a^2) = -x_0 b^2 (x - x_0)$$

$$\frac{(y - y_0) y_0}{b^2} = \frac{-x_0 (x - x_0)}{a^2}$$

$$\frac{y \cdot y_0}{b^2} - \frac{y_0^2}{b^2} = \frac{-x \cdot x_0}{a^2} + \frac{x_0^2}{a^2}$$

$$\frac{y \cdot y_0}{b^2} + \frac{x \cdot x_0}{a^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2}$$

$$\boxed{\frac{y \cdot y_0}{b^2} + \frac{x_0 x_0}{a^2} = 1}$$

(14) $y^q = x^p$

$q y^{q-1} \cdot y' = p x^{p-1}$

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$$y' = \frac{p}{q} \frac{x^{p-1}}{y^{q-1}}$$

$$= \frac{p}{q} \frac{x^{p-1}}{(y^q)(y^{-1})}$$

$$= \frac{p}{q} \frac{x^{p-1}}{x^p (x^{p/q})^{-1}}$$

$$= \frac{p}{q} \frac{x^{p-1} x^{p/q}}{x^p}$$

$$= \frac{p}{q} x^{-1} \cdot x^{p/q}$$

$$= \boxed{\frac{p}{q} x^{p/q - 1}}$$

(15)

Skipped.

16 $f(t) = 0.01t^4 - 0.04t^3$

(a) $\boxed{f'(t) = 0.04t^3 - 0.12t^2}$

(b) $f'(1) = 0.04 - 0.12$

$= \boxed{-0.08}$

(c) $f'(t) = 0.04t^3 - 0.12t^2 = 0$

$t^2(0.04t - 0.12) = 0$

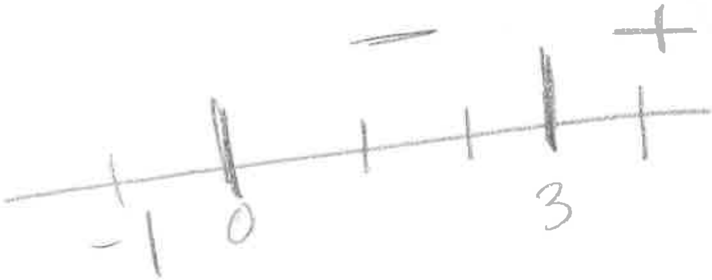
$t^2 = 0 \qquad 0.04t - 0.12 = 0$

$\boxed{t=0}$ and $\qquad 0.04t = 0.12$

$\boxed{t = 3}$

(d) $f'(t) > 0$

$t^2(0.04t - 0.12) > 0$



$\boxed{(3, \infty)}$

(e) $|f(0) - f(3)| + |f(3) - f(6)|$

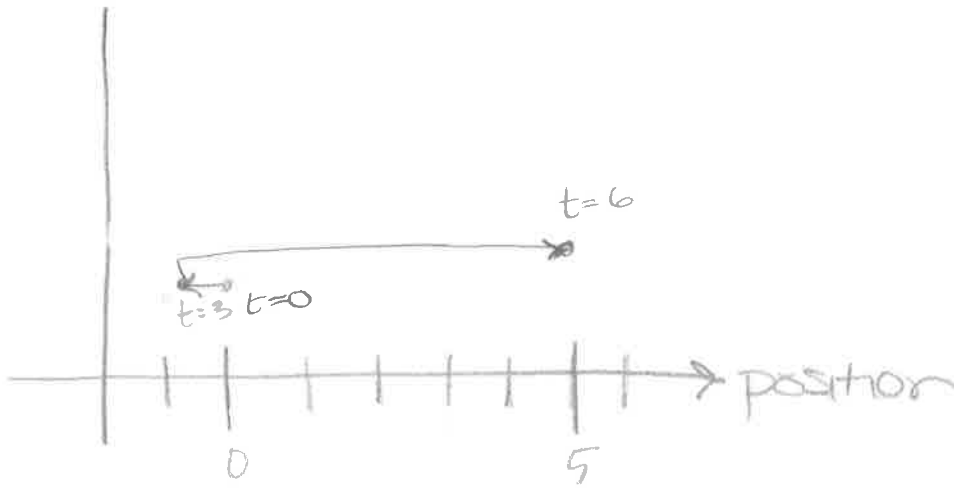
$f(0) = 0$

$f(3) = (0.01)(81) - 0.04(27) = 0.81 - 1.08 = -0.27$

$f(6) = (0.01)(1296) - 0.04(216) = 12.96 - 8.64 = 4.32$

$|f(0) - f(3)| + |f(3) - f(6)| = 0.27 + 4.32 = \boxed{4.59}$

(f)

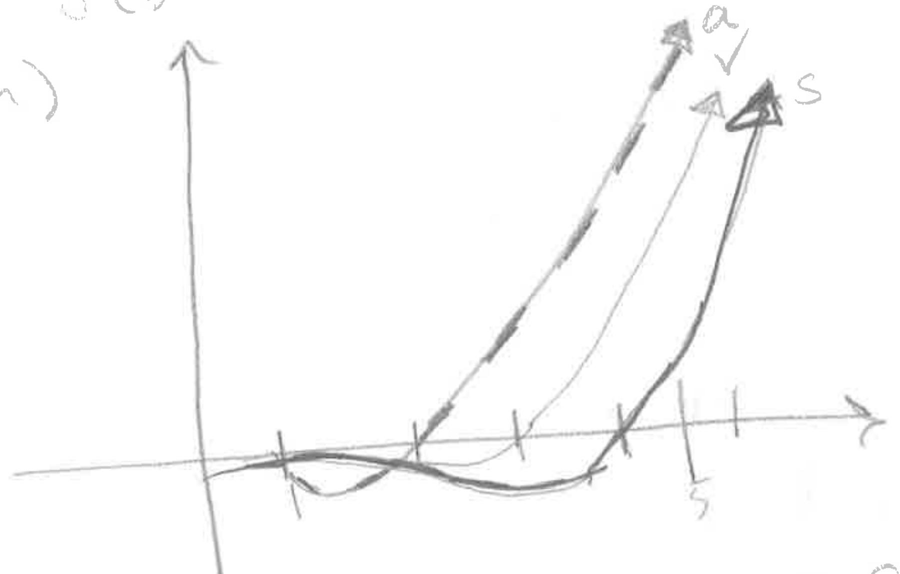


(g) $f'(t) = 0.04t^3 - 0.12t^2$

$\boxed{f''(t) = 0.12t^2 - 0.24t}$

$f''(1) = 0.12 - 0.24 = \boxed{-0.12}$

(h)



Used Wolfram Alpha

(i) Speeding Up: $f'(x) > 0$ & $f''(x) > 0$
or $f'(x) < 0$ & $f''(x) < 0$

Slowing Down: $f'(x) > 0$ & $f''(x) < 0$
or $f'(x) < 0$ & $f''(x) > 0$

Speeding Up: $(0,2) \cup (3,\infty)$

Slowing down: $(2,3)$

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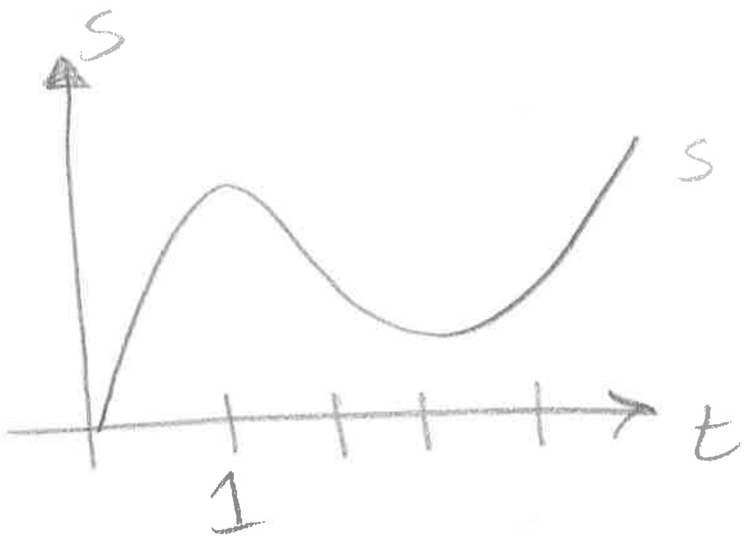
(a)



$f'(t) > 0$: $[0, 1) \cup (3, 4]$

$f'(t) < 0$: $(1, 3)$

$f''(t) > 0$: $(2, 4]$

$f''(t) < 0$: $[0, 2)$

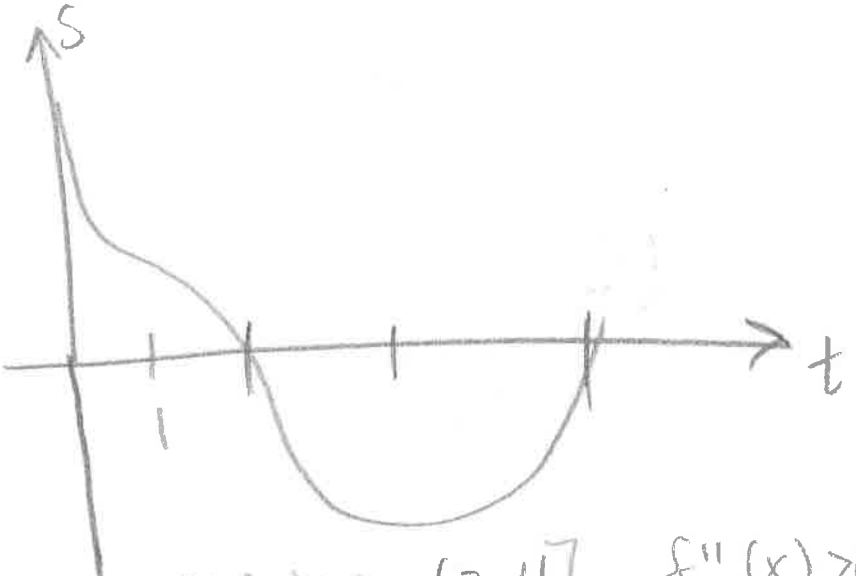
$f' < 0$ & $f'' < 0$ $\quad$ $f' > 0$, $f'' > 0$

Speeding up: $(1, 2) \cup (3, 4]$

Slowing down: $[0, 1) \cup (2, 3)$

$f' > 0$, $f'' < 0$ $\quad$ $f' < 0$, $f'' > 0$

(b)



$f'(x) > 0$ $(3, 4]$ $\quad$ $f''(x) > 0$ $[0, 1.5) \cup (2, 4]$

$f'(x) < 0$ $[0, 3)$ $\quad$ $f''(x) < 0$ $(1.5, 2)$

Speeding Up: $(1.5, 2)^{\leftarrow f'<0,\ f''<0} \cup (3, 4]$ $\uparrow f'>0 \ \& \ f''>0$

Slowing Down: $[0, 1.5) \cup (2, 3)$

$f'<0, f''>0$ $\quad$ $f'<0, f''>0$

(18) $s = t^4 - 4t^3 - 20t^2 + 20t \quad t \geq 0$

(a) $s' = 4t^3 - 12t^2 - 40t + 20$

$20 = 4t^3 - 12t^2 - 40t + 20$

$0 = 4t^3 - 12t^2 - 40t$

$0 = 4t(t^2 - 3t - 10)$

$0 = 4t(t-5)(t+2)$

At $t = 0$ & $t = 5$

(b) $s'' = 12t^2 - 24t - 40$

$0 = 12t^2 - 24t - 40$

$0 = 4(3t^2 - 6t - 10)$

$t = \dfrac{6 \pm \sqrt{36 - 4 \cdot 3 \cdot (-10)}}{2 \cdot 3}$

$t = \dfrac{6 \pm \sqrt{36 + 120}}{6}$

$\boxed{t = \dfrac{6 \pm \sqrt{156}}{6}}$

Speed is constant

(19) (a) (1) ✓

(2) 

(3) A - area
r - radius
t - time

(4) $\frac{dr}{dt}$ - given $\frac{dA}{dt}$ - required

(5) $A = \pi r^2$

(6) $\boxed{\frac{dA}{dt} = 2\pi r \frac{dr}{dt}}$

(b) (1) ✓ (2) 

(3) A - area
r - radius
t - time

(4) $\frac{dr}{dt} = 1 \, m/s$ $\frac{dA}{dt}$ - required at $r = 30m$.

(5) $A = \pi r^2$ (6) $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

(7) $\frac{dA}{dt} = 2\pi (30m) \cdot \frac{1m}{s}$

$= \boxed{60\pi \; m^2/s}$

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① ✓

②

p.185 #4

③ A - area
l - length
w - width
t - time

④ $\frac{dl}{dt} = 8 \text{ cm/s}$ $\quad \frac{dw}{dt} = 3 \text{ cm/s}$

Want: $\frac{dA}{dt}$ when $l = 20\text{cm}$ $\quad w = 10\text{cm}$

⑤ $A = lw$

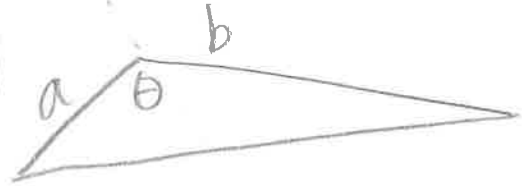
⑥ $\frac{dA}{dt} = w\frac{dl}{dt} + l\frac{dw}{dt}$

⑦ $\frac{dA}{dt} = 10\text{cm}\left(8\frac{\text{cm}}{\text{s}}\right) + 20\text{cm}\left(3\frac{\text{cm}}{\text{s}}\right)$

$= 80\frac{\text{cm}^2}{\text{s}} + 60\frac{\text{cm}^2}{\text{s}}$

$\boxed{\frac{dA}{dt} = 140\frac{\text{cm}^2}{\text{s}}}$

(21) (a) (1) ✓

(2) 

(3) a - side 1
b - side 2
θ - contained angle of a & b
A - area
t - time

(4) $\frac{d\theta}{dt} = 0.2\,\text{rad/min}$

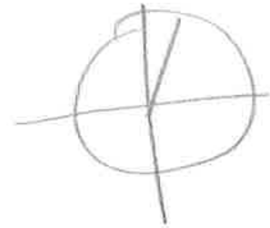
Want: $\frac{dA}{dt}$ when $\theta = \pi/3$, $a = 2\text{cm}$, $b = 3\text{cm}$

(5) $A = \frac{1}{2}ab\sin\theta$

Note: a & b are fixed! (they are constants)

(6) $\frac{dA}{dt} = \frac{1}{2}ab\cos\theta \frac{d\theta}{dt}$

(7) $\frac{dA}{dt} = \frac{1}{2}(2\text{cm})(3\text{cm})\cos(\frac{\pi}{3})\, 0.2\frac{\text{rad}}{\text{min}}$

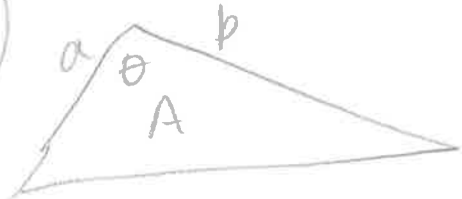


$= 3\text{cm}^2\,(\frac{1}{2})\,0.2\,\text{rad/min}$

$= \boxed{0.3\frac{\text{cm}^2\cdot\text{rad}}{\text{min}}}$

$1.5 \times .2 = 30$

(b) ① ✓ ②



③ a - side 1
b - side 2
θ - contained angle
A - area
t - time

④ $\frac{db}{dt} = 1.5\frac{cm}{min}, \frac{d\theta}{dt} = 0.2\frac{rad}{min}$

Want: $\frac{dA}{dt}$ when $b = 3cm$ & $\theta = \pi/3$, $a = 2cm$

⑤ $A = \frac{1}{2}ab\sin(\theta)$

⑥ $\frac{dA}{dt} = \frac{1}{2}a\sin(\theta)\frac{db}{dt} + \frac{1}{2}ab\cos(\theta)\frac{d\theta}{dt}$

⑦ $\frac{dA}{dt} = \frac{1}{2}(2cm)\sin\left(\frac{\pi}{3}\right)\left(1.5\frac{cm}{min}\right) + \frac{1}{2}(2cm)(3cm)\cos\left(\frac{\pi}{3}\right)0.2\frac{rad}{min}$

$\boxed{\frac{dA}{dt} = \frac{3\sqrt{3}}{\cancel{4}}\frac{cm^2 rad}{min} + \frac{6}{20}\frac{cm^2 rad}{min}}$

(c) ① ✓ ②

③ a - side 1
b - side 2
A - area
θ - contained angle
t - time

④ $\frac{da}{dt} = 2.5\frac{cm}{min}$ $\frac{db}{dt} = 1.5\frac{cm}{min}$ $\frac{d\theta}{dt} = 0.2\frac{rad}{min}$

want $\frac{dA}{dt}$ when $a = 2\text{cm}$ $b = 3\text{cm}$ $\theta = \pi/3$

⑤ $A = \frac{1}{2}ab\sin\theta$

⑥ $\frac{dA}{dt} = \frac{1}{2}b\sin\theta\frac{da}{dt} + \frac{1}{2}a\sin\theta\frac{db}{dt} + \frac{1}{2}ab\cos(\theta)\frac{d\theta}{dt}$

⑦ $\frac{dA}{dt} = \frac{1}{2}(3\text{cm})\sin\left(\frac{\pi}{3}\right)\left(2.5\frac{\text{cm}}{\text{min}}\right) + \frac{1}{2}(2\text{cm})\sin\left(\frac{\pi}{3}\right)\left(1.5\frac{\text{cm}}{\text{min}}\right) + \frac{1}{2}(2\text{cm})(3\text{cm})\cos\left(\frac{\pi}{3}\right)\cdot\left(0.2\frac{\text{rad}}{\text{min}}\right)$

$\frac{dA}{dt} = (1.5\text{cm})\left(2.5\frac{\text{cm}}{\text{min}}\right)\left(\frac{\sqrt{3}}{2}\right) + (1\text{cm})\frac{\sqrt{3}}{2}\left(1.5\frac{\text{cm}}{\text{min}}\right) + 1\text{cm}(3\text{cm})\frac{1}{2}\cdot .2\frac{\text{rad}}{\text{min}}$

$= \boxed{(3.75)\left(\frac{\sqrt{3}}{2}\right)\frac{\text{cm}^2\,\text{rad}}{\text{min}} + (1.5)\left(\frac{\sqrt{3}}{2}\right)\frac{\text{cm}^2\cdot\text{rad}}{\text{min}} + .3\frac{\text{cm}^2\,\text{rad}}{\text{min}}}$

㉒ $4x^2 + 9y^2 = 36$

$8x\frac{dx}{dt} + 18y\frac{dy}{dt} = 0$

(a) $8\cdot 2\frac{dx}{dt} + 18\cdot\frac{2}{3}\sqrt{5}\cdot\frac{1}{3} = 0$

$16\frac{dx}{dt} = \frac{-36}{9}\sqrt{5}$

$\boxed{\frac{dx}{dt} = \frac{-36}{9\cdot 16}\sqrt{5}}$

(b) $8x\frac{dx}{dt} + 18y\frac{dy}{dt} = 0$

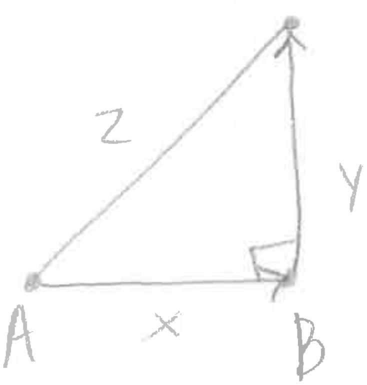
$$\begin{array}{r} \overset{1}{16} \\ 3 \\ \hline 48 \end{array}$$

$-16(3) + 18\left(\frac{2}{3}\sqrt{5}\right)\frac{dy}{dt} = 0$

$\frac{36}{3}\sqrt{5}\,\frac{dy}{dt} = -48$

$$\boxed{\frac{dy}{dt} = \frac{-48 \cdot 3}{36\sqrt{5}}}$$

23 ① ✓

② 

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#16

③ x - distance A is from starting point
for B
y - distance B has traveled
z - distance between A & B
t - time

④ $\frac{dx}{dt} = -35\,km/h$ $\frac{dy}{dt} = 25\,km/h$

Want: dz/dt at 4:00 PM !!

(5) $x^2+y^2=z^2$

(6) $2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$

$$\frac{dz}{dt}=\frac{1}{2z}\left(2x\frac{dx}{dt}+2y\frac{dy}{dt}\right)$$

(7) At 4:00 PM Ship A has traveled

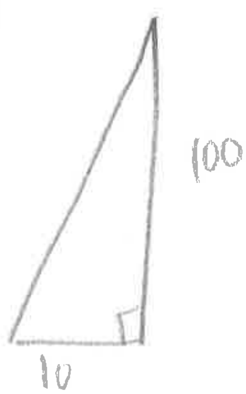
$$\begin{array}{r} {}^{2}35 \\ 4 \\ \hline 140 \end{array}$$

$$35\frac{km}{h}\cdot 4h = 140\ km$$

Ship B: $25\frac{km}{h}\cdot 4h = 100\ km$

So: $x = 150-140 = 10\ km$

$y = 100\ km$



$$z^2 = 10^2+100^2 = 10100$$

$$z=\sqrt{10100}$$

$$\boxed{\frac{dz}{dt}=\frac{1}{2\sqrt{10100}}\left(2\cdot10\left(-35\frac{km}{h}\right)+2\cdot\left(100\frac{km}{h}\right)\right)}$$

24 ① ✓

②

distance to 2nd:

③ x - distance to 1st

y - distance to 2nd

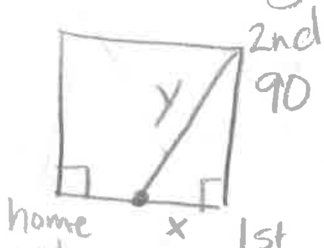
z - distance to 3rd

t - time

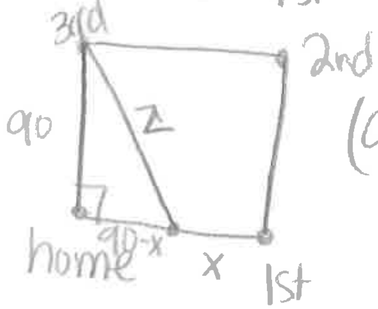
④ $\frac{dx}{dt} = -24\frac{ft}{s}$ want $\frac{dy}{dt}$ & $\frac{dz}{dt}$ when $x = \frac{1}{2}90$

$x = 45ft$

⑤



$x^2 + 90^2 = y^2$



$(90-x)^2 + 90^2 = z^2$

⑥ $2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ $\quad 2(90-x)\frac{dx}{dt} = 2z\frac{dz}{dt}$

$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$ $\quad \frac{dz}{dt} = \frac{(90-x)}{z}\frac{dx}{dt}$

⑦ $\frac{dy}{dt} = \frac{x}{y}\left(-24\frac{ft}{s}\right)$ $\frac{dz}{dt} = \frac{(90-x)}{z}\left(-24\frac{ft}{s}\right)$

When $x = 45ft$: $45^2 + 90^2 = y^2$

$2025 + 8100 = y^2$

$10{,}125 = y^2$

$y = \sqrt{10125}$

$(90-45)^2 + 90^2 = z^2$

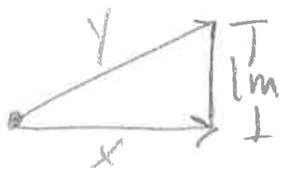
$45^2 + 90^2 = z^2$

$z = \sqrt{10125}$

$\boxed{\frac{dy}{dt} = \frac{45}{\sqrt{10125}}\left(-24\frac{ft}{s}\right)}$ $\boxed{\frac{dz}{dt} = \frac{45}{\sqrt{10125}}\left(-24\frac{ft}{s}\right)}$

25 ① ✓

②



③ x - distance from boat to dock

y - amount of pulley out

t - time

④ $\frac{dy}{dt} = -1 m/s$ want: $\frac{dx}{dt}$ when $x = 8m$

(5) $x^2+(1m)^2=y^2$

(6) $2x\frac{dx}{dt}=2y\frac{dy}{dt}$

$\frac{dx}{dt}=\frac{y}{x}\frac{dy}{dt}$

(7) $x=8m$ so

$(8m)^2+(1m)^2=y^2$

$16m^2+1m^2=y^2$

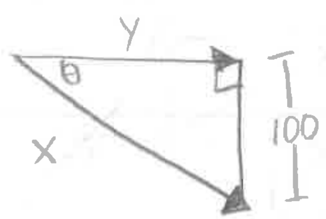
$y^2=17m^2$

$y=\sqrt{17}m$

$\boxed{\frac{dx}{dt}=-\frac{\sqrt{17}}{8}\frac{m}{s}}$

(26) (1) ✓

(2)



(3) x - amount of string
y - amount moved horizontally
t - time θ - angle

④ $\frac{dy}{dt} = 8 ft/s$ want $d\theta/dt$ when $x = 200 ft$

⑤ $\tan(\theta) = \frac{100}{y}$ $\tan(\theta) = 100y^{-1}$

⑥ $\sec^2(\theta)\frac{d\theta}{dt} = -100y^{-2}\frac{dy}{dt}$

$$\frac{d\theta}{dt} = \frac{-100}{y^2\sec^2(\theta)}\frac{dy}{dt}$$

When $x = 200 ft$: $x^2 = y^2 + 100^2$

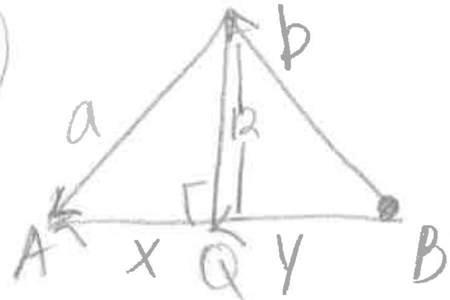
$$200^2 = y^2 + 100^2$$
$$y^2 = 40000 - 10000$$
$$y^2 = 30000$$
$$y = \sqrt{30000}$$

$\cos(\theta) = \frac{y}{x}$ $\cos(\theta) = \frac{\sqrt{30000}}{200} \rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{30000}}{200}\right)$

so $\boxed{\frac{d\theta}{dt} = \frac{-100}{(30000)\sec^2\left(\cos^{-1}\left(\frac{\sqrt{30000}}{200}\right)\right)}}$

27 1 ✓

2



3 x - distance A is from Q

y - distance B is from center

a - amount of pulley near A

b - amount of pulley near B

t - time

4 $\frac{da}{dt} = 2 ft/s$ want: $\frac{dy}{dt}$ when $x = 5ft$.

5 $x^2 + 12^2 = a^2$ & $y^2 + 12^2 = b^2$

6 Note: $\frac{da}{dt} = \frac{db}{dt}$ & $\frac{dx}{dt} = \frac{dy}{dt}$

$2x\frac{dx}{dt} = 2a\frac{da}{dt}$

$2x\frac{dy}{dt} = 2a\frac{da}{dt}$

$$\frac{dy}{dt} = \frac{a}{x}\frac{da}{dt}$$

when $x=5$: $5^2+12^2=a^2$

$25+144=a^2$

$169=a^2$

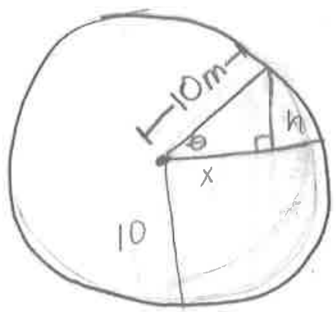
$a=13$

$$\frac{db}{dt} = \frac{13}{5}\left(\frac{2\,ft}{s}\right)$$

$$= \boxed{\frac{26}{5}\ ft/s}$$

(28) (1) ✓

(2)



(3) θ - angle, when measured like a unit circle

x - length of triangle

h - height of triangle

t - time

④ $\frac{d\theta}{dt} = \frac{2\pi \text{ rad}}{2 \text{ min}}$ $\quad \frac{dh}{dt}$ want $\quad$ when $h = 16m - 10m$

$h = 6m$

⑤ $\sin(\theta) = \frac{h}{10}$

⑥ $\cos(\theta)\frac{d\theta}{dt} = \frac{1}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = 10\cos(\theta)\frac{d\theta}{dt}$

⑦ when $h = 6m$

$\cos(\theta) = \frac{x}{10}$

$x^2 + 36 = 100$

$x^2 = 64$

$x = 8$

$\cos(\theta) = \frac{8}{10}$

$$\frac{dh}{dt} = 10 \cdot \frac{8}{10} \cdot \frac{2\pi}{2} \frac{rad}{min}$$

$$= 8\pi$$

29

30) $f(x) = \sin(x) \quad a = \pi/6$

$$L(x) = f'(a)(x-a) + f(a)$$

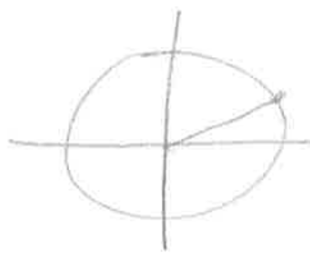
p.192

$\cancel{x}2$

$$f'(x) = \cos(x)$$

$$f'\left(\frac{\pi}{6}\right) = \cos(\pi/6)$$

$$= \frac{\sqrt{3}}{2}$$



$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = 1/2$$

$$\boxed{L(x) = \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}}$$

31) $f(x) = \dfrac{2}{\sqrt{x^2-5}} \quad a = 3$

$$L(x) = f'(a)(x-a) + f(a)$$

$$f(x) = \frac{2}{(x^2-5)^{1/2}} = 2(x^2-5)^{-1/2}$$

$$f'(x) = 2\left(-\frac{1}{2}\right)(x^2-5)^{-3/2}(2x)$$

$$= -2x(x^2-5)^{-3/2}$$

$$f'(3) = -2\cdot 3(3^2-5)^{-3/2}$$

$$= -6(9-5)^{-3/2}$$

$$= -6(4)^{-3/2}$$

$$= \frac{-6}{\sqrt{4^3}}$$

$$= \frac{-6}{\sqrt{64}}$$

$$= \frac{-6}{8}$$

$$= \frac{-3}{4}$$

$$f(3) = \frac{2}{\sqrt{9-5}} = \frac{2}{\sqrt{4}} = \frac{2}{2} = 1$$

$$\boxed{L(x) = \frac{-3}{4}(x-3)+1}$$

(32) $\sqrt{100.5}$

Let $f(x) = \sqrt{x}$ at $x = 100$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(100) = \frac{1}{2\cdot\sqrt{100}} = \frac{1}{2\cdot 10} = \frac{1}{20}$$

$$f(100) = \sqrt{100} = 10$$

$$L(x) = \frac{1}{20}(x-100) + 10$$

$$L(100.5) = \frac{1}{20}(100.5 - 100) + 10$$

$$L(100.5) = \frac{1}{20}(0.5) + 10$$

$$= \frac{1}{20}\cdot\frac{1}{2} + 10$$

$$= \frac{1}{40} + \frac{400}{40}$$

$$= \boxed{\frac{401}{40}}$$

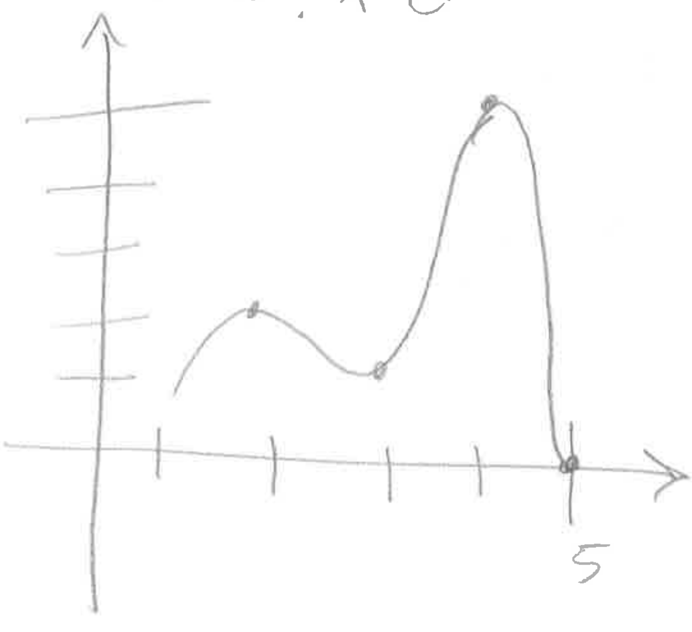
(33) (a) $\boxed{\text{Extreme Value Theorem}}$

(b) $\boxed{\text{Closed Interval Method}}$

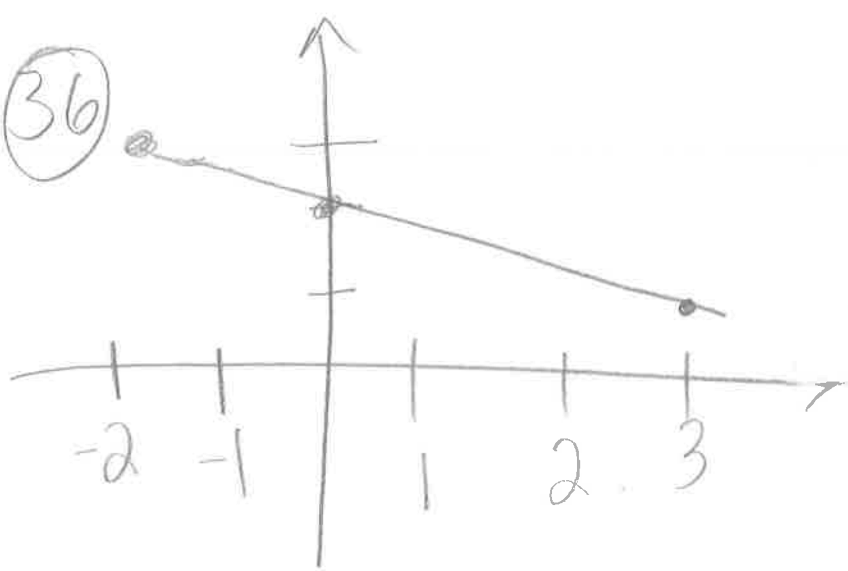
(34) absolute max : $x=r$
absolute min : $x=a$
local max : $x=b$, $x=r$
local min : $x=d$

(35)

p. 211 #8



(36)



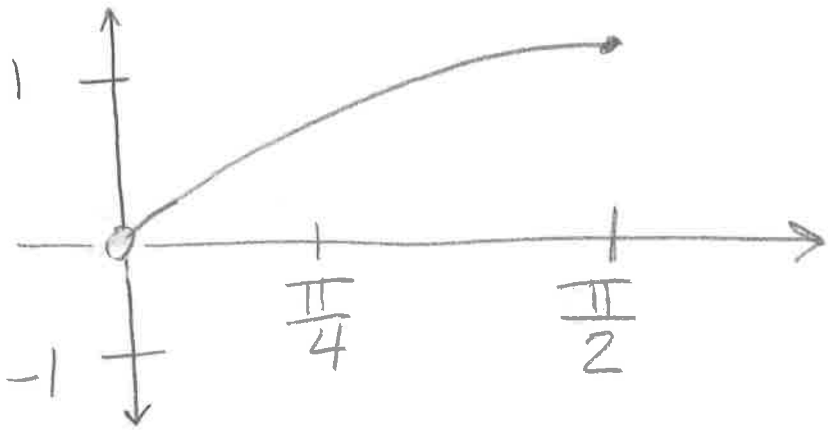
Absolute max: at $x=-2$

$f(-2) = 2 + \frac{2}{3} = \frac{6+2}{3} = \boxed{\frac{8}{3}}$

No min or local max/min.

p.32

(37) $f(x) = \sin(x) \quad 0 < x \leq \pi/2$



Absolute max: $f(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

No Absolute min or local max/min