1. (a)(i) $m = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1} \frac{x - x^3 - (1-1^3)}{x-1}$

$= \lim_{x \to 1} \frac{x - x^3 - 0}{x-1} = \lim_{x \to 1} \frac{x(1-x^2)}{x-1}$

$= \lim_{x \to 1} \frac{x(1-x)(1+x)}{x-1}$

$= \lim_{x \to 1} \frac{-x\cancel{(x-1)}(1+x)}{\cancel{(x-1)}} = \lim_{x \to 1} -x(1+x)$

$= -1(1+1) = -(2) = \boxed{-2}$

(ii) $m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{(1+h) - (1+h)^3 - 0}{h}$

$= \lim_{h \to 0} \frac{1+h - (1+3h+3h^2+h^3)}{h}$

$= \lim_{h \to 0} \frac{\cancel{1}+h-\cancel{1}-3h-3h^2-h^3}{h}$

$= \lim_{h \to 0} \frac{-2h-3h^2-h^3}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-2-3h-h^2)}{\cancel{h}}$

$= \lim_{h \to 0} -2-3h-h^2$

$= -2$

(b) $y - 0 = -2(x-1)$

$\boxed{y = -2(x-1)}$

2. $\lim_{x \to a} \frac{f(x) - f(a)}{x-a} = \lim_{x \to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x-a}$

$= \lim_{x \to a} \frac{\frac{\sqrt{a}}{\sqrt{xa}} - \frac{\sqrt{x}}{\sqrt{xa}}}{x-a}$

$\boxed{p.1}$

$$= \lim_{x\to a} \frac{\frac{\sqrt{a}-\sqrt{x}}{\sqrt{xa}}}{x-a} = \lim_{x\to a} \frac{\sqrt{a}-\sqrt{x}}{\sqrt{xa}} \div x-a$$

$$= \lim_{x\to a} \frac{\sqrt{a}-\sqrt{x}}{\sqrt{xa}} \cdot \frac{1}{(x-a)} = \lim_{x\to a} \frac{\sqrt{a}-\sqrt{x}}{\sqrt{xa}(x-a)}$$

$$= \lim_{x\to a} \left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{xa}(x-a)}\right)\cdot\left(\frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}}\right)$$

$$= \lim_{x\to a} \frac{a-x}{\sqrt{xa}(x-a)(\sqrt{a}+\sqrt{x})} = \lim_{x\to a} \frac{-\cancel{(x-a)}}{\sqrt{xa}\cancel{(x-a)}(\sqrt{a}+\sqrt{x})}$$

$$= \lim_{x\to a} \frac{-1}{\sqrt{xa}(\sqrt{a}+\sqrt{x})} = \frac{-1}{\sqrt{a^2}(\sqrt{a}+\sqrt{a})} = \frac{-1}{a(2\sqrt{a})}$$

$$= \boxed{\frac{-1}{2a\sqrt{a}}}$$

(b) m at (1,1) is: $\frac{-1}{2\cdot 1\sqrt{1}} = \frac{-1}{2\cdot 1} = \frac{-1}{2}$

$$\boxed{y-1 = \frac{-1}{2}(x-1)}$$

m at $(4,\frac{1}{2})$ is: $\frac{-1}{2\cdot 4\sqrt{4}} = \frac{-1}{8\cdot 2} = \frac{-1}{16}$

$$\boxed{(y-\tfrac{1}{2}) = \frac{-1}{16}(x-4)}$$

3. (a) $V(t) = S'(t)$ so

$$V(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}(t+h)^2 - 6(t+h) + 23 - (\frac{1}{2}t^2 - 6t + 23)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}(t^2+2th+h^2) - (6t+6h) + 23 - \frac{1}{2}t^2 + 6t - 23}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{\frac{1}{2}t^2} + th + \frac{1}{2}h^2 - \cancel{6t} - 6h + \cancel{23} - \cancel{\frac{1}{2}t^2} + \cancel{6t} - \cancel{23}}{h}$$

p.2

$$= \lim_{h\to 0} \frac{th + \frac{1}{2}h^2 - 6h}{h}$$

$$= \lim_{h\to 0} \frac{h(t+\frac{1}{2}h-6)}{h} = \lim_{h\to 0} t+\frac{1}{2}h-6$$

$$= t+6$$

$$v(t) = t - 6$$

(i) $\frac{s(8)-s(4)}{8-4} = \frac{7-7}{4} = \frac{0}{4} = 0$

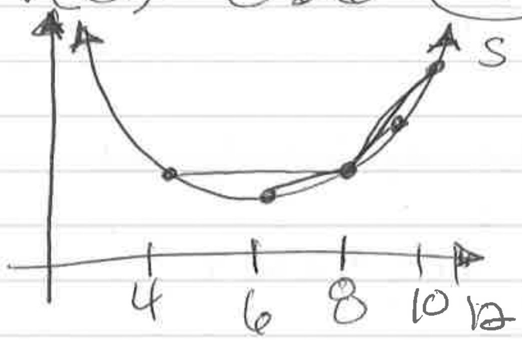
(ii) $\frac{s(8)-s(6)}{8-6} = \frac{7-5}{8-6} = \frac{2}{2} = 1$

(iii) $\frac{s(10)-s(8)}{10-8} = \frac{13-7}{2} = \frac{6}{2} = 3$

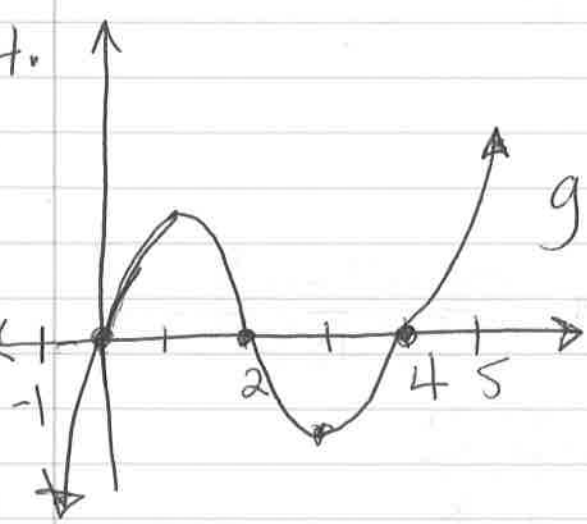
(iv) $\frac{s(12)-s(8)}{12-8} = \frac{23-7}{12-8} = \frac{16}{4} = 4$

(b) $v(8) = 8-6 = 2$

(c)



4.



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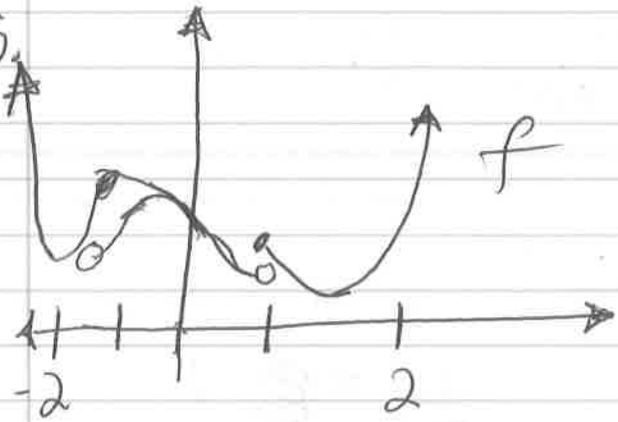


6. $f'(a) = \lim\limits_{t \to a} \dfrac{2t^3+t-(2a^3+a)}{t-a}$

$= \lim\limits_{t \to a} \dfrac{2t^3+t-2a^3-a}{t-a}$

$= \lim\limits_{t \to a} \dfrac{2t^3-2a^3+t-a}{t-a}$

$= \lim\limits_{t \to a} \dfrac{2(t-a)(t^2+at+a^2)+(t-a)}{t-a}$

$= \lim\limits_{t \to a} \dfrac{\cancel{(t-a)}(2(t^2+at+a^2)+1)}{\cancel{(t-a)}}$

$= \lim\limits_{t \to a} 2t^2+2at+2a^2+1$

$= 2a^2+2a^2+2a^2+1$

$= 6a^2+1$

x 7. $\lim\limits_{x \to a} \dfrac{\frac{4}{\sqrt{1-x}} - \frac{4}{\sqrt{1-a}}}{x-a} = \lim\limits_{x \to a} \dfrac{4\sqrt{1-a}-4\sqrt{1-x}}{\sqrt{(1-a)(1-x)}}$

$= \lim\limits_{x \to a} \dfrac{4(\sqrt{1-a}-\sqrt{1-x})}{\sqrt{(1-a)(1-x)}(x-a)} \cdot \dfrac{\sqrt{1-a}+\sqrt{1-x}}{\sqrt{1-a}+\sqrt{1-x}}$

$= \lim\limits_{x \to a} \dfrac{4((1-a)-(1-x))}{(\sqrt{(1-a)(1-x)})(\sqrt{1-a}+\sqrt{1-x})(x-a)}$

$$= \lim_{x \to a} \frac{4\cancel{(x-a)}}{\sqrt{(1-a)(1-x)}\left(\sqrt{1-a}+\sqrt{1-x}\right)\cancel{(x-a)}}$$

$$= \lim_{x \to a} \frac{4}{\sqrt{(1-a)(1-x)}\left(\sqrt{1-a}+\sqrt{1-x}\right)}$$

$$= \frac{4}{\sqrt{(1-a)(1-a)}\left(\sqrt{1-a}+\sqrt{1-a}\right)}$$

$$= \frac{4}{(1-a)\left(2\sqrt{(1-a)}\right)}$$

$$= \frac{4}{2(1-a)\sqrt{1-a}}$$

8 $$\lim_{t \to 4} \frac{10+\frac{45}{t+1} - \left(10+\frac{45}{4+1}\right)}{t-4}$$

$$= \lim_{t \to 4} \frac{10+\frac{45}{t+1} - 10 - \frac{45}{5}}{t-4}$$

$$= \lim_{t \to 4} \frac{\frac{45}{t+1} - 9}{t-4} = \lim_{t \to 4} \frac{\frac{45-9(t+1)}{t+1}}{t-4}$$

$$= \lim_{t \to 4} \frac{45-9t-9}{(t+1)(t-4)} = \lim_{t \to 4} \frac{36-9t}{(t+1)(t-4)}$$

$$= \lim_{t \to 4} \frac{9(4-t)}{(t+1)(t-4)} = \lim_{t \to 4} \frac{-9\cancel{(t-4)}}{(t+1)\cancel{(t-4)}}$$

$$= \lim_{t \to 4} \frac{-9}{t+1} = \frac{-9}{4+1} = \frac{-9}{5}$$

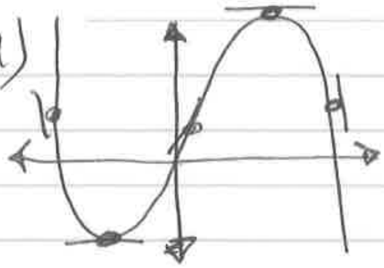
Velocity $= -9/5$

Speed $= 9/5$

9. (a) 1/4 (b) 0 (c) −1/2 (d) −1 (e) −1/2

(estimating slope) (f) −1/8 (g) 0 (h) 1/8

10. (a)



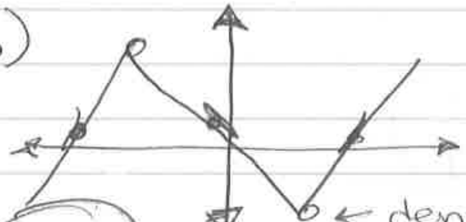
two points of slope zero

neg → zero → positive → zero → neg

− 0 + −

II

(b)

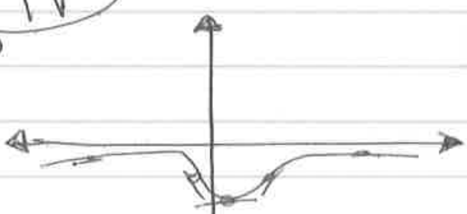


← derivative DNE

constant pos. → DNE → constant neg. → DNE → constant pos.

IV

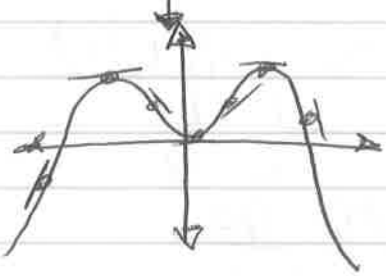
(c)



close to zero → neg → zero → positive → close to zero

I

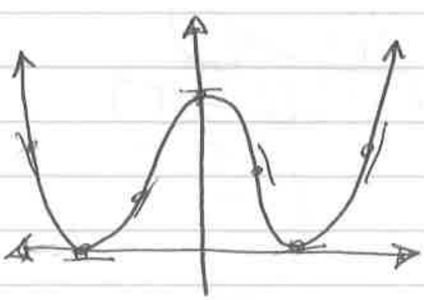
(d)



pos → zero → neg → zero → pos → zero → neg

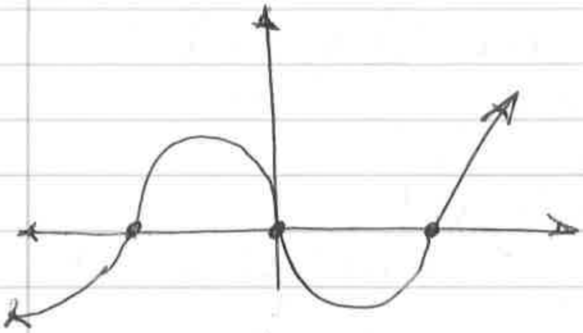
III

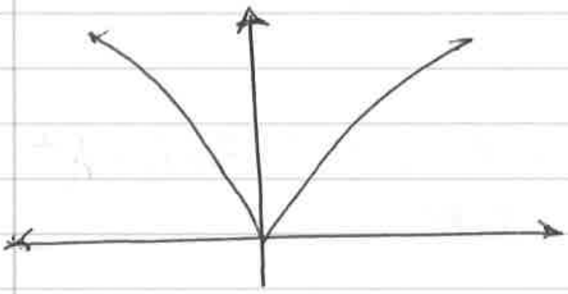
~~11.~~ 12.



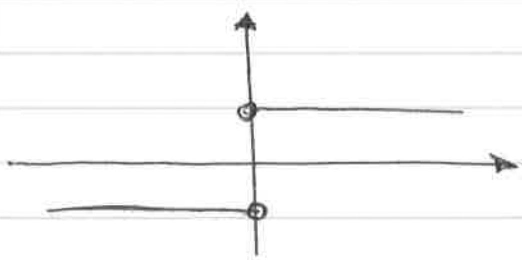
neg → zero → pos → zero → neg → zero → positive

p. 6

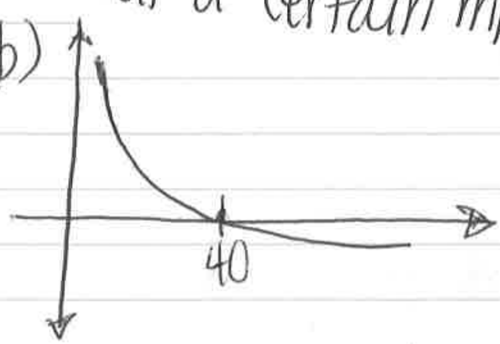


13. 

neg → DNE → pos



14. (a) $F'(v)$ indicates the rate at which the cost increases at a certain mph.

(b) 

(c) ≈35/40 mph.

15. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{m(x+h)+b-(mx+b)}{h}$

$= \lim_{h\to 0} \frac{\cancel{mx}+mh\cancel{+b}-\cancel{mx}\cancel{-b}}{h} = \lim_{h\to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h\to 0} m = m$

Domain of f: $(-\infty,\infty)$ Domain of f': $(-\infty,\infty)$

16. $f'(x) = \lim\limits_{h \to 0} \dfrac{(x+h)^{3/2} - x^{3/2}}{h} = \lim\limits_{h \to 0} \dfrac{\sqrt{(x+h)^3} - \sqrt[2]{x^3}}{h}$

$= \lim\limits_{h \to 0} \dfrac{\sqrt{(x+h)^3} - \sqrt{x^3}}{h} \cdot \dfrac{\sqrt{(x+h)^3} + \sqrt{x^3}}{\sqrt{(x+h)^3} + \sqrt{x^3}}$

$= \lim\limits_{h \to 0} \dfrac{(x+h)^3 - x^3}{h(\sqrt{(x+h)^3} + \sqrt{x^3})} = \lim\limits_{h \to 0} \dfrac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$

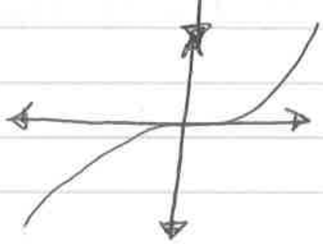
$= \lim\limits_{h \to 0} \dfrac{3x^2h + 3xh^2 + h^3}{h}$

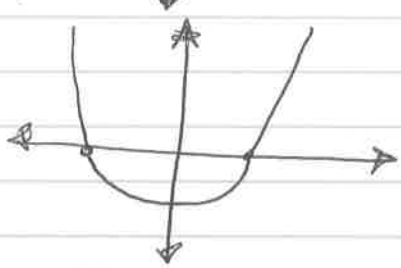
$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}} = \lim\limits_{h \to 0} 3x^2 + 3xh + h^2$

$= 3x^2$

17. a' looks like  b?

b' looks like  a?

C' looks like  b?

d' looks like  c?

So the only way it works out is:

$f = d \quad f' = c \quad f'' = b \quad f''' = a$

Another way to view this is that these look like x^2, x^3, x^4, & x^5.

18. (a) $\lim\limits_{x \to 0} \dfrac{x^{2/3} - 0}{x - 0} = \lim\limits_{x \to 0} \dfrac{x^{2/3}}{x} = \lim\limits_{x \to 0} x^{-1/3}$

$= \lim\limits_{x \to 0} \dfrac{1}{\sqrt[3]{x}}$ DNE

Do not worry about (b) - (d) for Exam.
I'll update these solutions later.

19. $f'(0) = \boxed{0}$ (constant)

20. $g'(x) = \frac{7}{4} \cdot 2x - 3 = \boxed{\frac{7}{2}x - 3}$

21. $f'(t) = (1.4)5t^4 - (2.5) \cdot 2t$
$= 7.0t^4 - 5.0t$

22. $H'(u) = 3(u+2) + (3u-1)(1)$
$= 3u + 6 + 3u - 1$
$= 6u + 5$

23. $B'(y) = -6cy^{-7}$

24. $y' = \frac{5}{3}x^{2/3} - \frac{2}{3}x^{-1/3}$

25. $y = \sqrt[3]{x}(2+x) = x^{1/3}(2+x)$
$y' = \frac{1}{3}x^{-2/3}(2+x) + x^{1/3}(1)$
$= \frac{2}{3}x^{-2/3} + \frac{1}{3}x^{1/3} + x^{1/3}$
$= \frac{2}{3}x^{-2/3} + \frac{4}{3}x^{1/3}$

26. $S'(R) = 8\pi R$

27. $y = \dfrac{\sqrt{x} + x}{x^2} = \dfrac{x^{1/2} + x}{x^2}$

$$y' = \frac{\left(\frac{1}{2}x+1\right)x^2 - \left(x^{1/2}+x\right)2x}{x^4}$$

$$= \frac{\frac{1}{2}x^3 + x^2 - 2x^{3/2} + 2x^2}{x^4}$$

$$= \frac{\frac{1}{2}x^3 + 3x^2 - 2x^{3/2}}{x^4}$$

$$= \frac{x^{3/2}\left(\frac{1}{2}x^{3/2} + 3x^{1/2} - 2\right)}{x^4}$$

$$= \frac{\frac{1}{2}x^{3/2} + 3x^{1/2} - 2}{x^{1/2}}$$

$$= \frac{1}{2}x + 3 - \frac{2}{\sqrt{x}}$$

28. $G(t) = \sqrt{5t} + \dfrac{\sqrt{7}}{t} = \sqrt{5}\sqrt{t} + \sqrt{7}\cdot t^{-1} = \sqrt{5}t^{1/2} + \sqrt{7}t^{-1}$

$$G'(t) = \frac{\sqrt{5}}{2}t^{-1/2} - \sqrt{7}t^{-2}$$

29. $D'(t) = \dfrac{32t(4t)^3 - (1+16t^2)3(4t)^2\cdot 4}{\left((4t)^3\right)^2}$

$$= \frac{2048t^4 - 192t^2 - 3072t^4}{4096t^6}$$

30. $h'(t) = \dfrac{6(6t-1)-(6t+1)(6)}{(6t-1)^2}$

$= \dfrac{36t-6-36t-6}{(6t-1)^2}$

$= \dfrac{-12}{(6t-1)^2}$

31. $y' = 6x^2 - 2x \quad m \text{ at } (1,3) = 6 \cdot 1^2 - 2 \cdot 1$

$= 6 - 2$

$= 4$

$(y-3) = 4(x-1)$

32. $f'(x) = \dfrac{0(3-x)-(-1)}{(3-x)^2} = \dfrac{1}{(3-x)^2}$

$f''(x) = \dfrac{0(3-x)^2 - 2(3-x)(-1)}{(3-x)^4}$

$= \dfrac{2\cancel{(3-x)}}{\cancel{(3-x)^4}}$

$= \dfrac{2}{(3-x)^3}$

33. $S'(A) = (0.882)(0.842)A^{(0.842-1)}$

$= 0.742644\ A^{-0.158}$

$S'(100) = 0.742644\,(100)^{-0.158}$

≈ 0.35874

In area $A = 100$, the rate of growth of the # of trees is 0.35874.

34. $f'(x) = nx^{n-1}$ $f''(x) = n(n-1)\,x^{n-1-1} = n(n-1)\,x^{n-2}$
$f'''(x) = n(n-1)(n-2)\,x^{n-2-1} = n(n-1)(n-2)x^{n-3}$
so $f^{(k)} = n\cdot(n-1)\cdot\ldots(n-(k-1))\;x^{n-k}$

35. $f'(x) = \cos(x) + x\sin(x) + 2\sec^2(x)$

36. $y' = 2\sec(x)\tan(x) + \csc(x)\cot(x)$

37. $g'(t) = 4\sec(x)\tan(x) + \sec^2(x)$

38. $(a\cos(u) + b\cot(u)) + u(-a\sin(u) - b\csc^2(u))$

39. $y' = \cos^2\theta - \sin^2\theta$

40. $y' = \dfrac{-\sin x(1-\sin(x)) - \cos(x)(1-\sin(x))}{\sin^2(x)}$

41. $y' = \dfrac{\cos(t)(1+\tan(t)) - \sin(t)(\sec^2(t))}{(1+\tan(t))^2}$

42. $y' = 2x(\sin(x)\tan(x)) + x^2(\cos(x)\tan(x) + \sin(x)\sec^2(x))$

43. I will get to this after Exam 2.

44. $f'(t) = \sec(x)\tan(x)$
$f''(t) = \sec(x)\tan^2(x) + \sec^3(x)$
$f''(\frac{\pi}{4}) = \frac{2}{\sqrt{2}}\cdot 1 + \left(\frac{2}{\sqrt{2}}\right)^3$

45. $\lim_{x \to 0} \frac{\sin(x)}{\sin(\pi x)} \cdot \frac{1/\pi x}{1/\pi x} = \lim_{x \to 0} \frac{\frac{\sin(x)}{\pi x}}{\frac{\sin(\pi x)}{\pi x}} \Rightarrow$

$= \lim_{\pi x \to 0} \frac{\left(\frac{1}{\pi}\right)\frac{\sin(x)}{x}}{\frac{\sin(\pi x)}{\pi x}} = \frac{1}{\pi}$

46. $\lim_{x \to 0} \frac{\sin(3x)\sin(5x)}{x^2} = \lim_{x \to 0} \frac{\sin(3x)}{x} \cdot \frac{\sin(5x)}{\cancel{x}}$

$= \lim_{x \to 0} \frac{\sin(3x)}{3x} \cdot 3 \cdot \lim_{x \to 0} \frac{\sin(5x)}{5x} \cdot 5$

$= 3 \cdot 5$

$= 15$