Homework 3 Solutions

1. Study Guide, page 15 #5 C, D and F

(C) This is already in standard form, so the center and radius can be read off immediately.

$\boxed{\text{Center: } (0, \frac{5}{6}) \text{ and Radius: } \frac{9}{11}}$

(D) I complete the square:

$$\begin{aligned} &x^2 + y^2 + 16x - 22y - 40 = 0 \\ &x^2 + 16x + y^2 - 22y = 40 \\ &x^2 + 16x + 64 + y^2 - 22y + 121 = 40 + 64 + 121 \\ &(x+8)^2 + (y-11)^2 = 225 \end{aligned}$$

Now we are in standard form for a circle so the center and radius can be read off immediately as $\boxed{\text{Center: } (-8, 11) \text{ and Radius: } 15}$.

(F) I complete the square:

$$\begin{aligned} &x^2 + y^2 + 11x - 36y - 86.75 = 0 \\ &x^2 + 11x + \frac{121}{4} + y^2 - 36y + 324 = 86.75 + \frac{121}{4} + 324 \\ &(x + \frac{11}{2})^2 + (y-18)^2 = 86.75 + 30.25 + 324 \\ &(x+5.5)^2 + (y-18)^2 = 441 \end{aligned}$$

Now we are in standard form for a circle so the center and radius can be read off immediately as $\boxed{\text{ Circle: } (-5.5, 18) \text{ and Radius: } 21}$.

2. Study Guide, page 15, #6

I find the midpoint of the two endpoints of the diameter to get the center of the circle

$$\begin{aligned} (h,k) &= \left(\frac{-4+8}{2}, \frac{18+34}{2} \right) \\ &= \left(\frac{4}{2}, \frac{52}{2} \right) \\ &= (2, 26) \end{aligned}$$

Next I find the distance from the center to an endpoint of the diameter to find the radius of the circle:

$$\begin{aligned} r &= \sqrt{(-4-2)^2 + (18-26)^2} \\ &= \sqrt{(-6)^2 + (-8)^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \qquad = 10 \end{aligned}$$

With the center and radius I can write my equation of a circle in standard form as

$$\boxed{(x-2)^2 + (y-26)^2 = 100}$$

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3. Study Guide, page 17 #3 (a), (b), and (d)
(a) A horizontal line passes through the y-axis, so has all the same y values. Therefore, my line is $\boxed{y = -4}$.
(b) A vertical line passes through the x-axis and thus has all the same x coordinates, so the line is $\boxed{x = -1}$.
(d) A vertical line with x-intercept of 9 must pass through the point (9,0), since this is what it means to have an x-intercept of 9. Therefore, as before, our vertical line must be $\boxed{x = 9}$.

4. Study Guide, page 16 #1 (d) and (e)
(d) To find an x-intercept, we must plug in $y = 0$ to the equation:

$$\frac{1}{2}x - \frac{1}{4}(0) = 9$$
$$\frac{1}{2}x = 9$$
$$x = 18$$

So our x-intercept is at $\boxed{(18,0)}$. To find the y-intercept, I plug in $x = 0$. Therefore,

$$\frac{1}{2}(0) - \frac{1}{4}y = 9$$
$$-\frac{1}{4}y = 9$$
$$y = -36$$

So our y-intercept is at $\boxed{(0,-36)}$.
(e) I find the x-intercept by plugging in $y = 0$:

$$x^2 - 0^3 = 8$$
$$x^2 = 8$$
$$x = \sqrt{8} \text{ and } x = -\sqrt{8}$$

So we have two x-intercepts and they are $\boxed{(2\sqrt{2},0) \text{ and } (-2\sqrt{2},0)}$. Similarly, for the y-intercepts, we plug in $x = 0$ and find that

$$0^2 - y^3 = 8$$
$$-y^3 = 8$$
$$y^3 = -8$$
$$y = -2$$

We only have one y-intercept and it is at $\boxed{(0, -2)}$.

5. Find the x-intercept(s) and y-intercept(s), if any, for each of the following:
(a) $3\sqrt{x} - 1 = y$
To find any x-intercepts I plug in $y = 0$:

$$3\sqrt{x} - 1 = 0$$
$$3\sqrt{x} = 1$$
$$\sqrt{x} = \frac{1}{3}$$
$$x = \frac{1}{9}$$

So we have an x-intercept at $\boxed{(\frac{1}{9}, 0)}$. We find the y-intercept by calculating:

$$3\sqrt{0} - 1 = y$$
$$-1 = y$$

so my y-intercept is at $\boxed{(0, -1)}$.
(b)

x :	1	2	0	3	4
y :	5	0	13	3	2

The x-intercepts is all values where $y = 0$, thus the x-intercept is $\boxed{(2, 0)}$. Similarly, the y-intercept is at $\boxed{(0, 13)}$.
(c)

x :	1	8	0	3	4
y :	5	0	6	3	2

Same as before. The x-intercept is at $\boxed{(8, 0)}$ and the y-intercept is at $\boxed{(0, 6)}$.
6. Study Guide, page 16, #2 (c) (e) and (f)
(c) I check for x-axis symmetry by plugging in $-y$:

$$-y = 5x$$

This can not be simplified further, but it is not my original equation, so it is

$$\boxed{\text{not x-axis symmetric}}$$

. I check for y-axis symmetry by plugging in $-x$

$$y = 5(-x)$$
$$y = -5x$$

Again, this is not my original equation, so this is

$$\boxed{\text{not y-axis symmetric}}$$

. Finally, I check for origin symmetry by plugging in $-x$ and $-y$:

$$\begin{aligned} -y &= 5(-x) \\ -y &= -5x \\ y &= 5x \end{aligned}$$

I get to the last step above by multiplying both sides by (-1). Thus

$$\boxed{\text{yes, this is origin symmetric}}$$

, since this is my original equation.
(e) Checking for x-axis symmetry:

$$\begin{aligned} 3x(-y) &= 11 \\ -3xy &= 11 \end{aligned}$$

So

$$\boxed{\text{this is not x-axis symmetric}}$$

. Checking for y-axis symmetry:

$$\begin{aligned} 3(-x)y &= 11 \\ -3xy &= 11 \end{aligned}$$

This is not my original equation, so

$$\boxed{\text{no this is not y-axis symmetry}}$$

. Finally, I check for origin symmetry:

$$3(-x)(-y) = 11 \qquad 3xy = 11$$

This is my original equation, so

$$\boxed{\text{yes this is origin symmetry}}$$

.
(f) I check for x-axis symmetry:

$$-y = x^2 - 2$$

$$\boxed{\text{no this is not x-axis symmetric}}$$

. I check for y-axis symmetry:

$$y = (-x)^2 - 2$$
$$y = x^2 - 2$$

yes this is y-axis symmetric. I check for origin symmetry:

$$-y = (-x)^2 - 2$$
$$-y = x^2 - 2$$

no this is not origin symmetric.

7. Study Guide, page 19, # 4
I use the vertical line test and find that two points on the graph hit the same vertical line. Therefore, no this is not a function.

8. Is the following a function? Justify your answer.

x:	3	4	-1	6	3	9
y:	9	-1	0	1	-9	18

There is a repeated x value, so no this is not a function.

9. State if the following are functions. Justify your answer:
(a) $x = 3$
This is a vertical line, so by the Vertical Line test, no this is not a function.
(b) $y = 3$
This is a horizontal line, so by the Vertical Line test, yes this is a function.
(c) $y^2 + x^2 = 4$
I plug in $x = 0$ and see that:

$$y^2 + 0^2 = 4$$
$$y^2 = 4$$
$$y = 2 \text{ and } y = -2$$

Thus one value of x gives two values of y, so no this is not a function.
(d) $y = x^2 - 1$
I plug in a value of x, say $x = 0$ and get $y = 0^2 - 1$, which gives $y = -1$, just one answer. So yes this is a function.

10. Study Guide, page 19 #5
The domain is all the possible x-values I can plug in, which are all x-values where my graph exists, which is $[-6, 6]$. The range is all the possible y-values I can get, so its all the y-values

where my graph exists which are $\boxed{[0,6]}$.

11. Study Guide, page 18 #1 (A), (b), (C), (D), and (E)
(A) The domain is all the x-values: $\boxed{\{1,-4,0\}}$. The range is all the y-values: $\boxed{\{2,7,\pi\}}$.
(b) The domain is all the inputs: $\boxed{\{5,7\}}$. The range is all the outputs $\boxed{\{8\}}$.
(C) The domain is all the x-values: $\boxed{\{3,4,-1,6,9\}}$. The range is all the y-values: $\boxed{\{9,0,1,-9,18\}}$
(D) The domain is all possible numbers I can plug in for x. There are no restrictions, so my domain is $\boxed{(-\infty,\infty)}$. The range is all possible numbers I can get for y. Note that y can never be negative, so the range is $[0,\infty)$.
(E) The domain is all possible numbers I can plug in for x. There are no restrictions, so my domain is $\boxed{\text{all real numbers}}$. The range is all possible numbers I can get for y. Since the range of $y = x^2$ is $[0,\infty)$, the range of this equation is $[11,\infty)$.

12. Study Guide, page 20 #3 (A), (B), and (C)
(A) Underneath a square root all numbers must be greater than or equal to 0, therefore:

$$\begin{aligned}&3x - 12 \geq 0\\&3x \geq 12\\&x \geq 4\\&\boxed{[4,\infty)}\end{aligned}$$

(B) Underneath a square root must be greater than or equal to 0. A denominator cannot be zero, so

$$\begin{aligned}&2x + 10 > 0\\&2x > -10\\&x > -5\\&\boxed{(-5,\infty)}\end{aligned}$$

(C) A denominator cannot be zero. In this case, this is when

$$\begin{aligned}&4x - 2 = 0\\&4x = 2\\&x = \frac{1}{2}\end{aligned}$$

So I must take this number out of my domain. My domain is $\boxed{(-\infty,\frac{1}{2})\cup(\frac{1}{2},\infty)}$

13. Identify the domain of the following functions:
(a) $y = \frac{7}{x^2+x-20}$
I factor the bottom and get

$$y = \frac{7}{(x-4)(x+5)}$$

so the denominator equals 0 when $x = 4$ and $x = -5$ so my domain is all $x \neq \{4, -5\}$. I write this in interval notation by $\boxed{(-\infty, -5) \cup (-5, 4) \cup (4, \infty)}$.

(b) $y = \frac{7}{x^2+x-12}$

I factor the bottom and get

$$y = \frac{7}{(x+4)(x-3)}$$

so the denominator equals 0 when $x = -4$ or $x = 3$ so my domain is all $x \neq \{-4, 3\}$. I write this in interval notation by $\boxed{(-\infty, -4) \cup (-4, 3) \cup (3, \infty)}$.

14. Identify the range of $y = x^2 + 10$

The range of $y = x^2$ is $[0, \infty)$, so the range of $y = x^2 + 10$ is $\boxed{[10, \infty)}$.

15. Study Guide, page 20, #1

To calculate $f(0)$, wherever I see x I replace with 0 to get

$$f(0) = 2(0) + 5$$
$$f(0) = 0 + 5$$
$$\boxed{f(0) = 5}$$

Likewise:

$$f(-2) = 2(-2) + 5$$
$$f(-2) = -4 + 5$$
$$\boxed{f(-2) = 1}$$

$$f(k) = 2(k) + 5$$
$$\boxed{f(k) = 2k + 5}$$

$$f(t+1) = 2(t+1) + 5$$
$$f(t+1) = 2t + 2 + 5$$
$$\boxed{f(t+1) = 2t + 7}$$

$$f(x-2) = 2(x-2) + 5$$
$$f(x-2) = 2x - 4 + 5$$
$$\boxed{f(x-2) = 2x + 1}$$

$$f(x+h) = 2(x+h) + 5$$
$$\boxed{f(x+h) = 2x + 2h + 5}$$

16. Study Guide, page 20, #2

$$f(2) = 5(2) - 2^2$$
$$f(2) = 10 - 4$$
$$\boxed{f(2) = 6}$$

$$f(-3) = 5(-3) - (-3)^2$$
$$f(-3) = -15 - 9$$
$$\boxed{f(-3) = -24}$$

$$f(2k) = 5(2k) - (2k)^2$$
$$\boxed{f(2k) = 10k - 4k^2}$$

$$f(-3k^2) = 5(-3k^2) - (-3k^2)^2$$
$$\boxed{f(-3k^2) = -15k^2 - 9k^4}v$$

$$f(2x+h) = 5(2x+h) - (2x+h)^2$$
$$f(2x+h) = 10x + 5h - (2x+h)(2x+h)$$
$$f(2x+h) = 10x + 5h - (4x^2 + 4xh + h^2) \quad \boxed{f(2x+h) = 10x + 5h - 4x^2 - 4xh - h^2}$$

17. Study Guide, page 21, #4
To evaluate $f(0)$ I find which interval 0 is in to determine which function of my piecewise function to use. The second interval, $-1 \le x < 1$ is the correct interval, so I plug in 0 to $f(x) = 2x^2$. Therefore, $f(0) = 2(0^2) = \boxed{0}$. Similarly, -1 is also in that interval, so

$$f(-1) = 2(-1)^2$$
$$\boxed{f(-1) = 2}$$

2 is in the last interval, so

$$f(2) = 5 - 2(2)$$
$$f(2) = 5 - 4$$
$$\boxed{f(2) = 1}$$

1 is also in the last interval, since $1 \geq 1$,

$$f(1) = 5 - 2(1)$$
$$f(1) = 5 - 2$$
$$\boxed{f(1) = 3}$$

$\frac{1}{2}$ is in the second interval so

$$2(\frac{1}{2})^2$$
$$= 2\frac{1}{4}$$
$$= \boxed{\frac{1}{2}}$$

Lastly, -3 is in the first interval, so

$$f(-3) = 3(-3) - 1$$
$$f(-3) = -9 - 1$$
$$\boxed{f(-3) = -10}$$