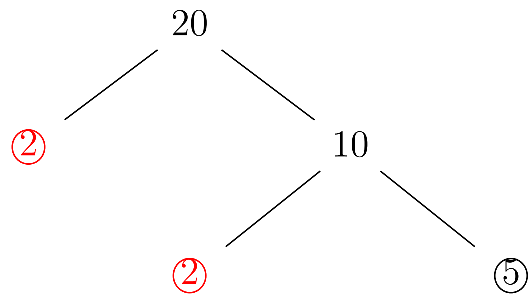
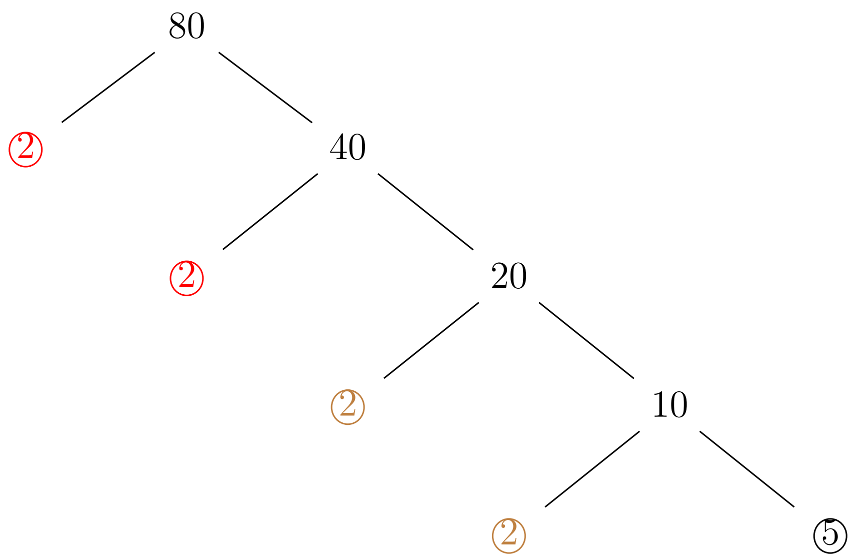
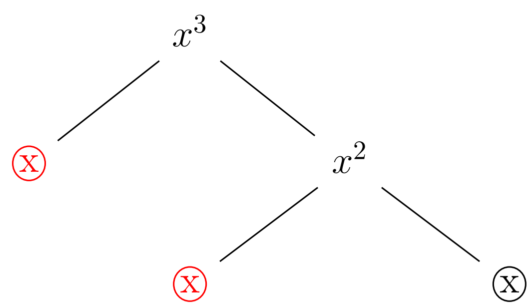
Homework 2 Solutions

1. Simplify:

$$5\sqrt{20x} + 3\sqrt{80x^3}$$







$$\begin{align}
&= 5 \cdot 2\sqrt{5x} + 3 \cdot 4\sqrt{5x^3} \tag{1}\\
&= 10\sqrt{5x} + 12\sqrt{5x^3} \tag{2}\\
&= 10\sqrt{5x} + 12x\sqrt{5x} \tag{3}\\
&= \boxed{(10 + 12x)\sqrt{5x}} \tag{4}
\end{align}$$

Note: I can NOT combine 10 and $12x$. One is a coefficient for x, the other is not.

2. What is the conjugate of $3 + 2\sqrt{5}$? $\boxed{3 - 2\sqrt{5}}$

3. Rationalize the denomiators:
(a) $\frac{1+2\sqrt{5}}{3-4\sqrt{7}}$

$$\begin{aligned}
&= \frac{1+2\sqrt{5}}{3-4\sqrt{7}} \cdot \frac{3+4\sqrt{7}}{3+4\sqrt{7}} \\
&= \frac{(1+2\sqrt{5})(3+4\sqrt{7})}{(3-4\sqrt{7})(3+4\sqrt{7})} \\
&= \frac{(1+2\sqrt{5})(3+4\sqrt{7})}{9+12\sqrt{7}-12\sqrt{7}-16(7)} \\
&= \frac{(1+2\sqrt{5})(3+4\sqrt{7})}{9-112} \\
&= \frac{(1+2\sqrt{5})(3+4\sqrt{7})}{-103} \\
&= \boxed{-\frac{(1+2\sqrt{5})(3+4\sqrt{7})}{103}}
\end{aligned}$$

(b) $\frac{12}{-\sqrt{15}}$

$$\begin{aligned}
&= \frac{12}{-\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} \\
&= \frac{12\sqrt{15}}{-15} \\
&= \boxed{-\frac{12\sqrt{15}}{15}}
\end{aligned}$$

4. Factor $x^2 - 3x - 10$ using any method.

I find two numbers that multiply to -10 and add to -3. I find these numbers to be -5 and 2. Thus I can factor to $\boxed{(x-5)(x+2)}$.

6. Factor:
(a) $x^2 - 121 = \boxed{(x-11)(x+11)}$
(b) $x^3 - 1331 = \boxed{(x-11)(x^2+11x+121)}$
(c) $512x^9 + 1$

$$\begin{aligned}
&= (8x^3+1)(64x^6 - 8x^3 + 1) \\
&= \boxed{(2x+1)(4x^2-2x+1)(64x^6-6x^3+1)}
\end{aligned}$$

(d) $9x^3 + 15x^2 - 12x - 20$

$$\begin{aligned}
&= (9x^3 + 15x^2)(-12x - 20) \\
&= 3x^2(3x+5) - 4(3x+5) \\
&= \boxed{(3x^2-4)(3x+5)}
\end{aligned}$$

7. Simplify:

$$\frac{x^2+4x+4}{x^2+6x+8}$$

$$\begin{aligned}
&= \frac{(x+2)(x+2)}{(x+4)(x+2)} \\
&= \boxed{\frac{(x+2)}{(x+4)}}
\end{aligned}$$

8. Simplify:

$$\frac{x^2-144}{x^2+6x} \div \frac{x^2-12x}{x^2-36}$$

$$\begin{aligned}
&= \frac{x^2-144}{x^2+6x} \cdot \frac{x^2-36}{x^2-12x} \\
&= \frac{(x-12)(x+12)}{x(x+6)} \cdot \frac{(x-6)(x+6)}{x(x-12)} \\
&= \frac{(x+12)}{x} \cdot \frac{(x-6)}{x} \\
&= \boxed{\frac{(x+12)(x-6)}{x^2}}
\end{aligned}$$

9. Find the complete solution set

$$(10-3x)^2 = 100$$

$$\begin{aligned}
&10 - 3x = 10 \\
&-3x = 0 \\
&\underline{x = 0}
\end{aligned}
\qquad\qquad
\begin{aligned}
&-(10-3x) = 10 \\
&-10 + 3x = 10 \\
&3x = 20 \\
&\underline{x = \frac{20}{3}}
\end{aligned}$$

Now I check my answers:

$$(10-3(0))^2 = 100$$
$$(10)^2 = 100$$
$$100 = 100$$

$$(10-3(\frac{20}{3})) = 100$$
$$(10-20)^2 = 100$$
$$(-10)^2 = 100$$
$$100 = 100$$

Both solutions work, so the final answer is $\boxed{x = 0 \text{ and } x = \frac{20}{3}}$.

10. Solve the equation for T:

$$Y = \frac{3A - 2B + 5T}{X} - 2$$

$$Y + 2 = \frac{3A - 2B + 5T}{X}$$
$$X(Y+2) = 3A - 2B + 5T$$
$$X(Y+2) - 3A + 2B = 5T$$
$$\boxed{\frac{X(Y+2) - 3A + 2B}{5} = T}$$

11. Find the complete solution set:
(a) $\sqrt{27-3x} = \sqrt{11-7x}$

$$27 - 3x = 11 - 7x$$
$$16 = -4x$$
$$\underline{-4 = x}$$

Plug into the original equation:

$$\sqrt{27-3(-4)} = \sqrt{11-7(-4)}$$
$$\sqrt{27+12} = \sqrt{11+28}$$
$$\sqrt{39} = \sqrt{39}$$

The solution works, so the answer is $\boxed{x = -4}$.
(b) $\sqrt{27-13x} = \sqrt{17-8x}$

$$27 - 13x = 17 - 8x$$
$$10 - 13x = -8x$$
$$10 = 5x$$
$$\underline{2 = x}$$

Plug back into the original equation:

$$\begin{aligned}\sqrt{27-13(2)} &= \sqrt{17-8(2)}\\ \sqrt{27-26} &= \sqrt{17-16}\\ \sqrt{1} &= \sqrt{1}\\ 1 &= 1\end{aligned}$$

The solution works so the answer is $\boxed{x=2}$.
(c) $|9-8x| = x$

$$\begin{aligned}&9-8x = x \\ &9 = 9x \\ &\underline{1 = x}\end{aligned}$$

$$\begin{aligned}&-(9-8x) = x\\ &-9+8x = x\\ &-9 = -7x\\ &\underline{\frac{9}{7} = x}\end{aligned}$$

Plug both answers into the original equation:

$$\begin{aligned}&|9-8(1)| = 1\\ &|9-8| = 1\\ &|1| = 1\\ &1 = 1\end{aligned}$$

$$\begin{aligned}&|9-8(\frac{9}{7})| = \frac{9}{7}\\ &|9-\frac{72}{7}| = \frac{9}{7}\\ &|\frac{63}{7}-\frac{72}{7}| = \frac{9}{7}\\ &|\frac{-9}{7}| = \frac{9}{7}\\ &\frac{9}{7} = \frac{9}{7}\end{aligned}$$

Both solutions work, so the answer is $\boxed{x = 1 \text{ and } x = \frac{9}{7}}$.

Note: In this problem all solutions worked, but in general that is not true. It is always important to check your answers, especially when I use the wording "find the complete solution set".

12. Find the complete solution set: (Hint: Square both sides and use the quadratic formula)

$$\sqrt{11x-28} = x$$

$$\begin{aligned}11x-28 &= x^2\\ 0 &= x^2-11x+28\\ 0 &= (x-7)(x-4)\end{aligned}$$

Then I have two possible solutions:

$$0 = x - 7 \qquad\qquad 0 = x - 4$$
$$\underline{7 = x} \qquad\qquad \underline{4 = x}$$

I check both answers by plugging into the original equation.

$$\sqrt{11(7) - 28} = 7 \qquad\qquad \sqrt{11(4) - 28} = 4$$
$$\sqrt{77 - 28} = 7 \qquad\qquad \sqrt{44 - 28} = 4$$
$$\sqrt{49} = 7 \qquad\qquad \sqrt{16} = 4$$
$$7 = 7 \qquad\qquad 4 = 4$$

Both answers are true so $\boxed{x = 7 \text{ and } x = 4}$.

13. Are the following True or False?
(a) $5 \geq 5$ $\boxed{\text{True}}$
(b) $5 > 4$ $\boxed{\text{True}}$

14. Find the complete solution set. Write your answer in interval notation.
(a) $8 - \frac{1}{10}x \geq -2$

$$-\frac{1}{10}x \geq -10$$
$$\underline{x \leq 100}$$

Next I check the value 0:

$$8 - \frac{1}{10}(0) \geq -2$$
$$8 \geq -2$$

This works, so my answer is indeed $x \leq 100$. I write this in interval notation by $\boxed{(-\infty, 100]}$.
(b) $-12 \leq 2x - 7 < 13$

$$-5 \leq 2x < 20$$
$$\underline{-\frac{5}{2} \leq x < 10}$$

I check a value that works for this interval, 0.

$$-12 \leq 2(0) - 7 < 13$$
$$-12 \leq -7 < 13$$

This is true so my answer is correct. I write it in interval notation by $\boxed{[-\frac{5}{2}, 10)}$.

15. Write the intervals in inequality notation:
(a) $(-\infty, 7) = \boxed{x < 7}$
(b) $(-1, 1) \cup (3, \infty) = \boxed{-1 < x < 1 \text{ or } x > 3}$

16. Study Guide, p. 14 #2 A
A. Call $x_1 = 3$, $y_1 = -4$, $x_2 = -2$, and $y_2 = 8$. Then the distance formula, which is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

gives that

$$\begin{aligned}
d &= \sqrt{(-2-3)^2 + (8-(-4))^2} \\
d &= \sqrt{(-5)^2 + 12^2} \\
d &= \sqrt{25 + 144} \\
d &= \sqrt{169} \\
&\boxed{d = 13}
\end{aligned}$$

17. Study Guide, p. 14 #3 I use the midpoint formula, which is

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

To find the midpoint for (A) I use the same numbering as in #16 and plug them into the Midpoint Formula to get

$$\begin{aligned}
(x, y) &= \left(\frac{3 + (-2)}{2}, \frac{-4 + 8}{2}\right) \\
&= \left(\frac{1}{2}, \frac{4}{2}\right) \\
&= \boxed{\left(\frac{1}{2}, 2\right)}
\end{aligned}$$

For the points in part (B) I call $x_1 = -16$, $y_1 = 24$, $x_2 = -8$, and $y_2 = -10$ then plug these values into the Midpoint Formula to get:

$$\begin{aligned}
(x, y) &= \left(\frac{-16 + (-8)}{2}, \frac{24 + (-10)}{2}\right) \\
&= \left(\frac{-24}{2}, \frac{14}{2}\right) \\
&= \boxed{(-12, 7)}
\end{aligned}$$

18. Study Guide, p. 14 #4 From the Midpoint Formula, I have

$$\left(\frac{-5+x}{2}, \frac{17+y}{2}\right) = (8, 2)$$

This gives me two equations:

$$\begin{aligned} \frac{-5+x}{2} &= 8 \\ -5+x &= 16 \\ x &= 21 \end{aligned} \qquad\qquad \begin{aligned} \frac{17+y}{2} &= 2 \\ 17+y &= 4 \\ y &= -13 \end{aligned}$$

So $\boxed{B = (21, -13)}$.