Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. Circle or box your answers. Show all work. Check your answers.

Name: ______________________________

ID number: ______________________________

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
5	4	
6	4	
7	4	
8	4	
9	4	
10	4	
11	4	
12	4	
13	4	
14	4	
15	4	
16	6	
17	6	
18	6	
19	6	
20	6	
21	6	
22	6	
Total:	102	

1. (4 points) Find the solution to $x^2 - 3x < -2$

Solution: Move everything over $x^2 - 3x + 2 < 0$. Now I factor the quadratic as $(x-2)(x-1) < 0$. So the two roots are 1 and 2. Then since $a > 0$ we have that the function is less than zero between the two roots, so $\boxed{(1,2)}$.

2. (4 points) Find the solution to $x^2 + 7x \geq -10$

Solution: I move everything over: $x^2+7x+10 \geq 0$. Then I factor the quadratic as $(x+5)(x+2) \geq 0$. So my two roots are -5 and -2. Also note that $a > 0$. Then my function is greater than zero outside its roots. It is zero at its roots, so my final answer is $\boxed{(-\infty,-5] \cup [-2,\infty)}$

3. (4 points) Find the solution to $-x^2 + 3x + 4 < 0$

Solution: Using the quadratic formula I can find the two roots to be

$$
\begin{aligned}
x &= \frac{-3 \pm \sqrt{9 - 4(-1)(4)}}{2(-1)} \\
x &= \frac{-3 \pm \sqrt{9+16}}{-2} \\
x &= \frac{-3 \pm \sqrt{25}}{-2} \\
x &= \frac{-3 \pm 5}{-2} \\
x &= \frac{-3+5}{-2} & x &= \frac{-3-5}{-2} \\
x &= \frac{2}{-2} & x &= \frac{-8}{-2} \\
x &= -1 & x &= 4
\end{aligned}
$$

Also note that $a < 0$, so our function is negative outside of the two roots. Thus our final answer is $\boxed{(,-\infty,-1) \cup (4,\infty)}$.
NOTE: If you gave an interval in your final answer, even if incorrect, you were given full credit for this problem.

4. (4 points) For the following function, is it a polynomial? If yes, what is its degree, leading coefficient, and leading term?

$$f(x) = x^6 + 9x^4 - 3x + x^{-1} + 4$$

Solution: $\boxed{\text{No}}$ this is not a polynomial.

5. (4 points) For the following function, is it a polynomial? If yes, what is its degree, leading coefficient, and leading term?

$$g(x) = x^{15} + x^4 + \frac{1}{3}x$$

Solution: $\boxed{\text{Yes}}$ this is a polynomial. Its degree is $\boxed{15}$, its leading term is $\boxed{x^{15}}$, its leading coefficient is $\boxed{1}$.

6. (4 points) What is the remainder when $x - 4$ is divided by $p(x) = 2x^4 - 8x^3 - 14x^2 + 81x - 36$

Solution: I plug is $x = 4$ to the polynomial.

$$\begin{aligned}
p(4) &= 2(4^4) - 8(4^3) - 14(4^2) + 81(4) - 36 \\
p(4) &= 2(256) - 8(64) - 14(16) + 324 - 36 - 224 \\
p(4) &= 512 - 512 + 324 - 36 - 224 \\
p(4) &= 64
\end{aligned}$$

Thus the remainder is $\boxed{64}$.

7. (4 points) Is $x - 5$ a factor of $p(x) = x^4 + 5x^3 - 7x^2 - 29x + 30$?

Solution: I can find if its a factor by plugging in 5:

$$\begin{aligned}
p(5) &= 2 = (5)^4 + 5(5)^3 - 7(5)^2 - 39(5) - 30 \\
p(5) &= 625 + 625 - 825 - 175 - 195 - 30 \\
p(5) &= 25
\end{aligned}$$

Since this solution is not zero, this is $\boxed{\text{not a factor}}$.

8. (4 points) Simplify the following:

$$\sqrt{-49} \cdot \frac{\sqrt{63}}{7\sqrt{-4}}$$

Solution:

$$\sqrt{49}\sqrt{-1} \cdot \frac{\sqrt{9 * 7}}{7\sqrt{-1}\sqrt{4}} = 7i \cdot \frac{\sqrt{9}\sqrt{7}}{(7 * 2)i} = 7i \cdot \frac{3\sqrt{7}}{14i} = \boxed{\frac{3\sqrt{7}}{2}}$$

9. (4 points) Compute the following:
(a) $(3+5i)+(7+4i)$
(b) $(7+4i)-(8+2i)$
(c) $(a+bi)(c+di)$

Solution:
(a) $\boxed{10+9i}$
(b) $\boxed{-1+2i}$
(c) I foil:

$$ac+adi+bci+bdi^2 = ac+adi+bci+bd(-1) = ac+adi+bci-bd = \boxed{(ac-bd)+(ad+bc)i}$$

10. (4 points) Compute the following:
(a) $(3+5i)(7+4i)$
(c) $\frac{(1+3i)}{(2-4i)}$

Solution:
(a)

$$21+12i+35i+20i^2 = 21+47i+20(-1) = 21+47i-20 = \boxed{1+47i}$$

(b)

$$\frac{2+4i+6i+12i^2}{4-8i+8i-16i^2} = \frac{2+10i-12}{4+16} = \frac{-10+10i}{20} = \boxed{\frac{-1}{2}+\frac{1}{2}i}$$

11. (4 points) State the number of complex roots of the following polynomial. Full credit is only given if work is shown.

$$f(x) = 2x^2-2x+9$$

Solution: Here $A=2$, $B=-1$, and $C=9$. Then

$$\begin{aligned} B^2-4AC &= (-1)^2-4(2)(9) \\ &= 1-72 \\ &= -71 \end{aligned}$$

There are no real roots, but there are $\boxed{2}$ complex roots.

12. (4 points) Evaluate the following:
(a) i^4
(b) i^{21}

Solution:
(a) $\frac{4}{4} = 1$ with Remainder 0. Thus $i^4 = 1$.
(b) $\frac{21}{4} = 5.25$ so $i^{21} = i$

13. (4 points) If $h(x) = f \circ g(x) = 4x + \sqrt{2x+7}$ and $g(x) = 2x$, find $f(x)$.

Solution: I can rewrite $h(x)$ as $h(x) = 2(2x) + \sqrt{2x+7}$, so $\boxed{f(x) = 2x + \sqrt{x+7}}$

14. (4 points) In the following table of $f(x)$, find $f(3)$ and $f^{-1}(14)$.

input :	0	1	2	3	4
output :	3	10	14	5	7

Solution: $\boxed{f(3) = 5}$ and $\boxed{f^{-1}(14) = 2}$.

15. (4 points) Find $f^{-1}(7)$ of the following function:

$$f(x) = \frac{x^3 + 13}{11}$$

Solution:

$$7 = \frac{x^3 + 13}{11}$$
$$77 = x^3 + 13$$
$$64 = x^3$$
$$\boxed{4}$$

16. (6 points) Let $f(x) = 3x^5 - x^4 - 31x^3 - 11x^2 + 76x + 60$. List all possible rational roots of $f(x)$. Find 4 numbers, n_1, n_2, n_3, and n_4 such that $\frac{f(x)}{x-n_1}$, $\frac{f(x)}{x-n_2}$, $\frac{f(x)}{x-n_3}$ and $\frac{f(x)}{x-n_4}$ all have remainder 0. These numbers will be whole numbers less than, not equal to, 4. Full credit will only be given if work is shown.

Solution: The factors of 60 are: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60. The factors of 3 are: 1, 3. Thus the Rational Roots Theorem gives that all our rational roots should be:

$$\boxed{\pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm10, \pm12, \pm15, \pm20, \pm30, \pm60, \pm\frac{1}{3} \pm \frac{2}{3}, \pm\frac{4}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{20}{3}}$$

. Now I test these solutions:

$$f(1) = 3 - 1 - 31 - 11 + 76 + 60 = 2 - 42 + 136 = -40 + 136 = 96$$
$$f(-1) = -3 - 1 + 31 - 11 - 76 + 60 = -4 + 20 - 16 = 16 - 16 = 0$$
$$f(2) = 3 * 32 - 16 - 31 * 8 - 11 * 4 + 76 * 2 + 60 = 0$$
$$f(-2) = 3 * (-32) - 16 + 31 * 8 - 11 * 4 - 76 * 2 + 60 = 0$$
$$f(3) = 3 * 243 - 81 - 31 * 27 - 11 * 9 + 76 * 3 + 60 = 0$$

So my four solutions are $\boxed{-1, 2, -2, \text{ and } 3}$.

17. (6 points) For one of the values found in the question above, n_1, n_2, n_3, or n_4 please calculate $\frac{f(x)}{x-n_1}$, $\frac{f(x)}{x-n_2}$, $\frac{f(x)}{x-n_3}$ or $\frac{f(x)}{x-n_4}$ using Synthetic Division. If you did not get an answer to #16, please use the number $-\frac{5}{3}$. Keep in mind your remainder should be zero. Please write your result as a polynomial.

Solution:

	3	-1	-31	-11	76	60
-1		-3	4	27	-16	-60
	3	-4	-27	16	60	0

	3	-1	-31	-11	76	60
2		6	10	-42	-106	-60
	3	5	-21	-53	-30	0

	3	-1	-31	-11	76	60
-2		-6	14	34	-46	-60
	3	-7	-17	23	30	0

	3	-1	-31	-11	76	60
3		9	24	-21	-96	-60
	3	8	-7	-32	-20	0

	3	-1	-31	-11	76	60
$-\frac{5}{3}$		-5	10	35	-40	-60
	3	-6	-21	24	36	0

Thus the polynomial for -1 would be: $\boxed{3x^4 - 4x^3 - 27x^2 + 16x + 60}$.

The polynomial for 2 would be: $\boxed{3x^4 + 5x^3 - 21x^2 - 53x - 30}$.

The polynomial for -2 would be: $\boxed{3x^4 - 7x^3 - 17x^2 + 23x + 30}$.

The polynomial for 3 would be: $\boxed{3x^4 + 8x^3 - 7x^2 - 32x - 20}$.

The polynomial for $-\frac{5}{3}$ would be: $\boxed{3x^4 - 6x^3 - 21x^2 + 24x + 36}$

So my y-intercept is at $\boxed{(0, \dfrac{7}{25})}$.

NOTE: The problem only asked for ONE of these. Please read questions carefully to prevent yourself from doing extra work.

18. (6 points) Find the domain, x-intercept(s), y-intercept(s), horizontal asymptotes, and vertical asymptotes of the following rational function:

$$r(x) = \frac{5x-7}{9x^2-25}$$

Solution: Domain: $9x^2 - 25 = 0$ factors as $(3x-5)(3x+5) = 0$ so my two roots are $\frac{5}{3}$ and $-\frac{5}{3}$, so my domain is $\boxed{(-\infty, -\frac{5}{3}) \cup (-\frac{5}{3}, \frac{5}{3}) \cup (\frac{5}{3}, \infty)}$. To find the vertical asymptote, I notice that none of the factors on the denominator cancel, so I have two at $\boxed{x = -\frac{5}{3} \text{ and } x = \frac{5}{3}}$. To find the horizontal asymptotes, I notice that the degree of the polynomial on the denominator is larger than the degree of the polynomial of the numerator, so my horizontal asymptote is $\boxed{y = 0}$. I find the x-intercepts by setting $r(x) = 0$

$$\begin{aligned} 0 &= \frac{5x-7}{9x^2-25} \\ 0 &= 5x - 7 \\ x &= \frac{7}{5} \end{aligned}$$

So my x-intercept is at $\boxed{(\frac{7}{5}, 0)}$. To find my y-intercepts I set $x = 0$, so

$$\begin{aligned} r(0) &= \frac{5(0)-7}{9(0)^2-25} \\ r(0) &= \frac{-7}{-25} \\ r(0) &= \frac{7}{25} \end{aligned}$$

So my y-intercept is at $\boxed{\left(0, \frac{7}{25}\right)}$

19. (6 points) If the square of the variable Y varies directly with the cube root of X and varies inversely with the sum of the variable Z and the cube of the variable M and if when $X = 8$, $Z = 5$, $M = 1$, then $Y = 2$, find Y when $X = 27$, $Z = 16$, and $M = 3$.

Solution: I can write this equation as:

$$Y^2 = \frac{k\sqrt[3]{X}}{(Z + M^3)}$$

Now I find k:

$$\begin{aligned} 2^2 &= \frac{k\sqrt[3]{8}}{(5 + 1^3)} \\ 4 &= \frac{2k}{(5+1)} \\ 4 &= \frac{2k}{6} \\ 24 &= 2k \\ k &= 12 \end{aligned}$$

I can plug that into my original equation as:

$$Y^2 = \frac{12\sqrt[3]{X}}{(Z + M^3)}$$

and now solve for Y:

$$\begin{aligned} Y^2 &= \frac{8\sqrt[3]{27}}{(16 + 3^3)} \\ Y^2 &= \frac{12(3)}{16 + 27} \\ Y^2 &= \frac{37}{43} \\ &\boxed{Y = \pm\frac{6}{\sqrt{43}}} \end{aligned}$$

20. (6 points) Divide the two polynomials using polynomial long division

$$\frac{3x^4 + x^3 - 11x^2 - 3x + 6}{x^2 - 3}$$

Solution:

$$\begin{array}{r|rrrrr}
 & & & 3x^2 & +x & -2 \\ \hline
x^2-3 & 3x^4 & +x^3 & -11x^2 & -3x & +6 \\
 & -3x^4 & & +9x^2 & & \\ \cline{2-4}
 & & x^3 & -2x^2 & -3x & \\
 & & -x^3 & & +3x & \\ \cline{3-5}
 & & & -2x^2 & & +6 \\
 & & & 2x^2 & & -6 \\ \cline{4-6}
 & & & & & 0
\end{array}$$

21. (6 points) Find the following composition of functions when

$$f(x) = 2x + 1,\ g(x) = 4x^2 - 3x + 3, \text{ and } h(x) = \frac{x-1}{2}$$

(i) $(f \circ g)(0)$
(ii) $(f \circ h)(x)$
(iii) $(f \circ g \circ h)(3)$

Solution:
(i) $f(g(0)) = f(3) = \boxed{7}$
(ii) $f(h(x)) = 2h(x) + 1 = x - 1 + 1 = \boxed{x}$
(iii) $f(g(h(3))) = f(g(1)) = f(4 - 3 + 3) = f(4) = \boxed{9}$

22. (6 points) Find the inverse of the following function:

$$f(x) = \frac{5x-4}{2x-1}$$

Solution:

$$y = \frac{5x-4}{2x-1}$$
$$x = \frac{5y-4}{2y-1}$$
$$x(2y-1) = 5y - 4$$
$$2xy - x = 5y - 4$$
$$2xy - 5y = x - 4$$
$$(2x-5)y = x - 4$$
$$y = \frac{x-4}{2x-5}$$
$$\boxed{f^{-1}(x) = \frac{x-4}{2x-5}}$$