Name:______________________________

Math 3333 Section 002 — *Practice Exam 3* — *March 31, 2015*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let $V = \mathbb{R}^2$ the vector space of 2−columns. Let $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in V$ and show that $\langle \mathbf{x}, \mathbf{y} \rangle = x_1y_1 - x_2y_1 - x_1y_2 + 3x_2y_2$ defines an inner product on V.

2. Let $V = C[-\pi, \pi]$ the vector space of all continuous functions on $[-\pi, \pi]$. Let $f,\ g \in V$ and show that

$$\langle f, g \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)g(t)\ dt$$

defines an inner product on V.

3. Use the Gram-Schmidt process to find an orthonormal basis for $S = \{1, t\}$ the standard basis for P_1 with the following inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\ dt$$

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4. Use the Gram-Schmidt process to find an orthonormal basis for $S = \{1, t, e^t\}$ a basis for a subspace to $C[0, 1]$ with the following inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\ dt$$

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5. Let A and B be two $n \times n$ invertible matrices. Show that $\det(I_n - AB) = \det(I_n - BA)$.

6. Let A be a skew-symmetric $n \times n$ matrix. Show that if n is odd then $\det(A) = 0$.

7. Let $L : P_3 \to \mathbb{R}$ be the linear transformation defined by

$$L(a_1t^3 + a_2t^2 + a_3t + a_4) = \int_0^1 (a_1t^3 + a_2t^2 + a_3t + a_4)\ dt$$

Find a basis for $\ker(L)$ and compute $\dim \ker(L)$.

8. Let $L : P_2 \to P_2$ be the linear transformation defined by

$$L(a_1t^2 + a_2t + a_3) = t\frac{d}{dt}\left(a_1t^2 + a_2t + a_3\right)$$

Find a basis for $\ker(L)$ and compute $\dim \ker(L)$.

9. Let P be a $n \times n$ matrix whose columns are orthonormal. For $\mathbf{x} \in \mathbb{R}^n$, show that $\|P\mathbf{x}\| = \|\mathbf{x}\|$. HINT: If the columns of P are orthonormal than what is P^tP equal to?