Name:____________________________

Math 3333 Section 002 | *Practice Exam 2* | *February 25, 2015*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit! On the actual exam the last page will have a list of matrices and their row reduced echelon or row echelon forms that you may or may not need.

1. Let

$$C^1(\mathbb{R}) = \{f(x) : f'(x) \text{ exists}\}$$

Show that $C^1(\mathbb{R})$ is a subspace of $C(\mathbb{R})$, the set of all continuous functions on the real line under the usual addition of functions and scalar multiplication.

2. Let

$$C_0(\mathbb{R}) = \left\{ f(x) : \lim_{x \to \pm\infty} f(x) = 0 \right\}$$

Show that $C_0(\mathbb{R})$ is a subspace of $C(\mathbb{R})$, the set of all continuous functions on the real line under the usual addition of functions and scalar multiplication.

3. Let

$$S = \left\{t^2+1, t^2+t, t+1\right\}$$

Determine if the following set is a basis for P_2, the set of all polynomials of degree 2 or less.

4. Let

$$S = \left\{ \begin{pmatrix} 2 \\ -1 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 2 \\ 8 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \\ 5 \end{pmatrix} \right\}$$

Determine if the following set is a basis for $\mathbb{R}^4$, the set of all column vectors with 4 entries.

5. Find a basis and the dimension to the kernel of the following matrix

$$A = \begin{pmatrix} 1 & -1 & 2 & 1 & 0 \\ 2 & 0 & 1 & -1 & 3 \\ 5 & -1 & 3 & 0 & 3 \\ 4 & -2 & 5 & 1 & 3 \\ 1 & 3 & -4 & -5 & 6 \end{pmatrix}$$

6. Find a basis and the dimension to the kernel of the following matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & -1 \\ 2 & 3 & 2 & 0 \\ 3 & 4 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$$

7. Let A be an $n \times n$ matrix. Show that the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if the columns of A are linear independent. (HINT: You will need the rank-nullity theorem)

8. Let A be an $m \times n$ matrix. Let $A^t A$ be nonsingular. Show that $\text{rk}(A) = n$. (HINT: You will need the rank-nullity theorem)

9. Let A be an $n \times n$ matrix. Suppose there is no nonzero vector $\mathbf{x} \in \mathbb{R}^n$, such that $A\mathbf{x} = \mathbf{x}$. Show that $A - I_n$ is nonsingular. (HINT: You will need the rank-nullity theorem.)