Name:____________________________

Math 3333 Section 002 | *Practice Exam 1* | *January 30, 2015*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 4 & 5 & 7 \\ 2 & 1 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 5 & 1 \\ 1 & 8 & 1 \end{pmatrix}$$

Compute AB and $\mathrm{tr}(AB)$.

2. Let

$$A = \begin{pmatrix} 3 & 0 & 2 \\ 8 & -1 & 4 \\ 2 & 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 4 & 5 \\ -5 & 1 & 2 \\ 3 & 0 & 1 \end{pmatrix}$$

Compute AB and $\mathrm{tr}(AB)$.

3. Let A be a skew symmetric $n \times n$ matrix. Show that A^k is skew symmetric for odd k and is symmetric for even k.

4. Let A be any $n \times n$ matrix. Show that AA^t and A^tA are symmetric matrices.

5. Let A and B be nonsingular $n \times n$ matrices. Show that AB is nonsingular and

$$(AB)^{-1} = B^{-1}A^{-1}$$

6. Let A be a nonsingular $n \times n$ matrix. Show that A^t is nonsingular and

$$(A^t)^{-1} = (A^{-1})^t$$

7. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

8. Let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$$

It is known that A is invertible. Compute A^{-1} using row reduction.

9. Let A be an $n \times n$ matrix. Show that $tr(A^t A) \geq 0$.
HINT: If $A = [a_{ij}]$ and $B = [b_{ij}]$ then $AB = C = [c_{ij}]$ with

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$