Name:_____

Math 3113 *Practice Final Exam* *April 23, 2013*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

**1**. Solve the following differential equation:

$$y' = \frac{xe^{-y} - 1}{x + e^{y}}$$

**2**. Solve the following differential equation:

$$(x^2 + 1)y' + 3x^3 y = 6x \exp\left(-\frac{3}{2}x^2\right)$$

**3**. Solve the following differential equation:

$$xe^y y' = 2e^y + 2x^3 e^{2x}$$

**4**. Solve the following differential equation:

$$y'' - 4y = \sinh(2x)$$

**5**. Solve the following differential equation:

$$y'' + 9y = 2\sec(3x)$$

**6**. Solve the following differential equation:

$$y'' + y = \tan x$$

**7**. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 - 3x_2 \\ x_2' = 3x_1 + 4x_2 \end{cases}$$

**8**. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 + 2x_2 \\ x_2' = 3x_1 - x_2 \end{cases}$$

**9**. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 2x_1 + 3x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$$

**10**. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = 0, \quad y'(1) = 0 \end{cases}$$

**11**. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = 0, \quad y(1) = 0 \end{cases}$$

**12**. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + \lambda y = 0 \\ y(-\pi) = y(\pi), \quad y'(-\pi) = y'(\pi) \end{cases}$$

**13**. Find the inverse Laplace transform of the following function:

$$F(s) = \ln\left(\frac{s^2+1}{s^2+4}\right)$$

**14**. Find the inverse Laplace transform of the following function:

$$F(s) = \ln\left(1 + \frac{1}{s^2}\right)$$

**15**. Find the inverse Laplace transform of the following function:

$$F(s) = \frac{s}{(s^2+1)^3}$$

**16**. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' - 2x' + tx = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

Hint: $\sin\tau \sin(t-\tau) = \dfrac{1}{2}\left(\cos(2\tau - t) - \cos t\right)$

**17**. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + (t-2)x' + x = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

**18**. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + (3t-1)x' + 3x = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

**19**. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \delta(t) \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

**20**. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \delta(t) + \delta(t-\pi) \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

**21**. Compute the Laplace transform of the following function:

$$f(t) = 1 + [[t]]$$

on $[0, \infty)$, where $[[t]]$ is the greatest integer function.