1. $(x + e^y)^2 = x^2 + C$

2. $y(x) = 2\ \exp\left(-\frac{3}{2}x^2\right) + C\ \exp\left(-\frac{3}{2}x^2\right)\left(x^2+1\right)^{\frac{3}{2}}$

3. $y(x) = \ln\left(x^2 e^{2x} + Cx^2\right)$

4. $y(x) = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{4} x \sinh(2x) - \frac{1}{16}\sinh(2x)$

5. $y(x) = C_1\cos(3x) + C_2\sin(3x) + \frac{2}{9}\cos(3x)\ln(\cos(3x)) + \frac{2}{3}x\sin(3x)$

6. $y(x) = C_1\cos x + C_2 \sin x + \cos x \ln|\sec x + \tan x| - \cos^2 x - \sin(2x)$

7. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} -\sin(3t) \\ \cos(3t) \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} \cos(3t) \\ \sin(3t) \end{pmatrix} e^{4t}$

8. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + C_2 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-2t}$

9. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

10. All the eigenvalues, λ are $\lambda \geq 1$. For the eigenvalue $\lambda = 1$, the corresponding eigenfunction is $y(x) = xe^{-x}$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \alpha_n^2$ where α_n is a root to the following equation $\tan\alpha = \alpha$. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\alpha_n x)$ for $\lambda > 1$.

11. All the eigenvalues, λ are $\lambda > 1$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \pi^2 n^2$ where n is an integer. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\pi n x)$.

12. All the eigenvalues, λ are nonnegative. When $\lambda = 0$, the corresponding eigenfunction is just $y =$ constant. When $\lambda > 0$ one has $\lambda_n = -n^2$. The corresponding eigenfunctions are $y_n(x) = \cos(nx)$ and $y_n(x) = \sin(nx)$.

13. $\mathscr{L}^{-1}(F(s)) = \frac{2}{t}\left(\cos(2t) - \cos t\right)$

14. $\mathscr{L}^{-1}(F(s)) = 2 - \frac{2}{t} + \frac{2}{t}e^{-t}$

15. $\mathscr{L}^{-1}(F(s)) = \frac{t}{8}(\sin t - t\cos t)$

16. $x(t) = \frac{C}{2}\left(\sin t - t\cos t\right)$

17. $x(t) = \frac{C}{6}t^3 e^{-t}$

18. $x(t) = \dfrac{C}{2}t^2e^{-3t}$

19. $x(t) = \dfrac{1}{2}\sin(2t)$

20. $x(t) = \dfrac{1}{2}\sin(2t) + \dfrac{1}{2}\sin(2t-\pi)u(t-\pi)$

21. $\mathscr{L}\left(1 + [[t]]\right) = \dfrac{1}{s(1-e^{-s})}$