**1**. $\det(A) = -78$ so an inverse exists and $A^{-1} = \frac{-1}{78}\begin{pmatrix} 3 & -12 \\ -8 & 6 \end{pmatrix}$, $\det(B(t)) = 0$, so no inverse exists, $\det(C) = -43$ so an inverse exists and $C^{-1} = \frac{-1}{43}\begin{pmatrix} 1 & -2 \\ -32 & 21 \end{pmatrix}$

**2**. $\det(A) = 11$ so an inverse exists and $A^{-1} = \frac{1}{11}\begin{pmatrix} 3 & -2 \\ -8 & 9 \end{pmatrix}$, $\det(B(t)) = 2$, so an inverse exists and $(B(t))^{-1} = \frac{1}{2}\begin{pmatrix} e^{-3t} & e^{t} \\ -e^{-t} & e^{3t} \end{pmatrix}$, $\det(C(t)) = -22$ so an inverse exists and $(C(t))^{-1} = \frac{-1}{22}\begin{pmatrix} \frac{1}{t} & -4 \\ -7 & 6t \end{pmatrix}$

**3**. $A'(t) = \begin{pmatrix} -\sin t & \sec^2 t \\ \csc^2 t & \sec t \tan t \end{pmatrix}$, and $B'(t) = \begin{pmatrix} 30e^{5t} & 5t^4 \\ -5t^{-6} & -5e^{-5t} \end{pmatrix}$, finally $\det(AB) = 10$.

**4**. $A'(t) = \begin{pmatrix} e^t + te^t & \cos t \\ -\csc t \cot t & -6e^{-t} \end{pmatrix}$, and $B'(t) = \begin{pmatrix} 3e^t \ln t + \frac{3e^t}{t} & \frac{1}{t} \\ 0 & -e^{-t} \end{pmatrix}$, finally $\det(AB) = (2 - 12t)\ln t$.

**5**. $x(t) = c_1 e^{4t}\begin{pmatrix} 3 \\ 2 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

**6**. $x(t) = c_1 e^{4t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{3t}\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

**7**. $x(t) = c_1\begin{pmatrix} \cos(2t) - 2\sin(2t) \\ \cos(2t) \end{pmatrix} + c_2\begin{pmatrix} 2\cos(2t) + \sin(2t) \\ \sin(2t) \end{pmatrix}$

**8**. $x(t) = c_1\begin{pmatrix} \cos(3t) + \sin(3t) \\ -3\cos(3t) \end{pmatrix} + c_2\begin{pmatrix} -\cos(3t) + \sin(3t) \\ -3\sin(3t) \end{pmatrix}$

**9**. The eigenvalues are $\lambda_n = 1 + n^2\pi^2$ for nonzero interger $n$. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\pi n x)$.

**10**. All the eigenvalues, $\lambda_n$ are $\lambda_n \geq 1$. For eigenvalues $\lambda_n > 1$ one has $\lambda_n = 1 + \alpha_n^2$ where $\alpha_n$ is a nonzero root to the following equation $\tan\alpha = \alpha$. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\alpha_n x)$ When $\lambda_n = 1$, the corresponding eigenfunction is $y(x) = xe^{-x}$.

**11**. The eigenvalues are $\lambda_n = n^2$ for nonzero integer $n$. The corresponding eigenfunctions are $y_n(x) = \sin(nx)$ and $y_n(x) = \cos(nx)$.

**12**. The eigenvalues are $\lambda_1 = e^t + i$ and $\lambda_2 = e^t - i$. The eigenvector corresponding to $\lambda_1$ is $\mathbf{v}_1 = \begin{pmatrix} i \\ -e^t \end{pmatrix}$ and the eigenvector corresponding to $\lambda_2$ is $\mathbf{v}_2 = \begin{pmatrix} i \\ e^t \end{pmatrix}$