1. Just plug in to check

2. Just plug in to check

3. $f(x,y) = x^4 \ln y$ and $\dfrac{\partial f}{\partial y} = \dfrac{x^4}{y}$, and the point in question is $(1,1)$. Both f and $\partial f/\partial y$ are continuous at the point, therefore there exists a unique solution.

4. $f(x,y) = x^2 - y^7$ and $\dfrac{\partial f}{\partial y} = -7y^6$, and the point in question is $(0,1)$. Both f and $\partial f/\partial y$ are continuous at the point, therefore there exists a unique solution.

5. $f(x,y) = 1 + x^9 + y^4$ and $\dfrac{\partial f}{\partial y} = 4y^3$, and the point in question is $(0,2)$. Both f and $\partial f/\partial y$ are continuous at the point, therefore there exists a unique solution.

6. $y = \tan\left(C - x - \dfrac{1}{x}\right)$

7. $y = C \sin x$

8. $y = -1 + \dfrac{1}{C - \tan^{-1} x}$

9. $y = x^2 \sin x - 3x^2$

10. $y = \dfrac{1}{3} + \dfrac{16}{3}\left(x^2 + 4\right)^{-\frac{3}{2}}$

11. $y = 3xe^{2x}$

12. $(x + e^y)^2 = 2x^2 + C$

13. $y = \sqrt[3]{Ce^x - 3x^4 - 12x^3 - 36x^2 - 72x + 72}$

14. $y^2 = -4x^2 + (x \ln x + Cx)^2$

15. Just plug in to check