Name:_____

Math 3113 *Practice Exam 1* *January 29, 2013*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let C_1 and C_2 be constants. Let

$$y = C_1 x \cos(\ln x) + C_2 x \sin(\ln x)$$

Verify that y is a solution to the following differential equation:

$$x^2 y'' - xy' + 2y = 0$$

2. Let

$$y = \frac{1}{4}(x - \ln x) + \frac{3}{4}\left(\frac{1}{x} - \ln x\right)$$

Verify that y is a solution to the following differential equation:

$$x^2 y'' + xy' - y = \ln x$$

3. Given the following IVP:

$$\begin{cases} y' = x^4 \ln y \\ y(1) = 1 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

4. Given the following IVP:

$$\begin{cases} y' = x^2 - y^7 \\ y(0) = 1 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

5. Given the following IVP:

$$\begin{cases} y' = 1 + x^9 + y^4 \\ y(0) = 2 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

6. Solve the following differential equation:

$$x^2 y' = 1 - x^2 + y^2 - x^2 y^2$$

7. Solve the following differential equation:

$$y' \tan x = y$$

8. Solve the following differential equation:

$$y' + y'x^2 = (1+y)^2$$

9. Solve the following IVP:

$$\begin{cases} x\dfrac{dy}{dx} = 2y + x^3 \cos x \\ y\left(\dfrac{\pi}{2}\right) = \pi^2 \end{cases}$$

10. Solve the following IVP:

$$\begin{cases} (x^2+4)y' + 3xy = x \\ y(0) = 1 \end{cases}$$

11. Solve the following IVP:

$$\begin{cases} y' - 2y = 3e^{2x} \\ y(0) = 0 \end{cases}$$

12. Solve the following differential equation:

$$xy' + e^y y' = xe^{-y} - 1$$

13. Solve the following differential equation:

$$3y^2y' = 3x^4 + y^3$$

14. Solve the following differential equation:

$$xyy' = y^2 + x\sqrt{4x^2 + y^2}$$

15. Let C be a constant. Define $u(x)$ in the following way

$$u(x) = Ce^{x^2} + \int_0^x e^{x^2-t^2} \cosh t \; dt$$

Verify that u is a solution to the following differential equation:

$$u' - 2xu = \cosh x$$