Name:________________________

Math 3113 Section 003 | *Practice Final Exam* | *November 24, 2014*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Solve the following differential equation:

$$(1+x)y' + y = \cos x$$

2. Solve the following differential equation:

$$xy' = 2y + x^3 \cos x$$

3. Use variation of parameters to solve the following differential equation:

$$y'' - 4y = \sinh(2x)$$

4. Use variation of parameters to solve the following differential equation:

$$y'' + y = \tan x$$

5. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 - 3x_2 \\ x_2' = 3x_1 + 4x_2 \end{cases}$$

6. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 + 2x_2 \\ x_2' = 3x_1 - x_2 \end{cases}$$

7. Compute the Laplace transform of the following function $f(t) = (-1)^{[[t]]}$ for $t \geq 0$ where $[[t]]$ is defined to be the closest integer to t. (HINT: You will need the transform of a derivative of a function with discontinuities.)

8. Compute the Laplace transform of the unit staircase function $f(t) = 1 + [[t]]$ for $t \geq 0$ where $[[t]]$ is defined to be the closest integer to t. (HINT: You will need the transform of a derivative of a function with discontinuities.)

9. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \cos t \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

10. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x' + 8x = e^{-t} \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

11. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' - 2x' + tx = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

HINT: $\sin\tau \sin(t-\tau) = \dfrac{1}{2}\left(\cos(2\tau - t) - \cos t\right)$

12. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + (t-2)x' + x = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

13. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + x' + tx = 0 \\ x(0) = 1, \quad x'(0) = 0 \end{cases}$$

HINT: You should get

$$X(s) = \frac{C}{\sqrt{s^2+1}} = \frac{1}{s}\left(1 + \frac{1}{s^2}\right)^{-1/2}$$

Use the binomial series to expand the second term and compute the inverse term by term.