Name:____________________________

Math 3113 *Practice Exam 3* *October 28, 2014*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let

$$A = \begin{pmatrix} 5 & 2 \\ 9 & 4 \end{pmatrix}, B(t) = \begin{pmatrix} t & t^3 \\ t^2 & t^7 \end{pmatrix}$$

Compute $\det(A)$, $\det(B)$, and B^{-1} where it exists. Finally compute $\frac{dB}{dt}$.

2. Let

$$A = \begin{pmatrix} 7 & 5 \\ 1 & -2 \end{pmatrix}, B(t) = \begin{pmatrix} \ln t & t \\ \sin t & \cos t \end{pmatrix}$$

Compute $\det(A)$, $\det(B)$, and B^{-1} where it exists. Finally compute $\frac{dB}{dt}$.

3. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 + 2x_2 \\ x_2' = 3x_1 - x_2 \end{cases}$$

4. Find the general solution to the following first order system:

$$\begin{cases} x_1' = x_1 + 2x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$$

5. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 - 3x_2 \\ x_2' = 3x_1 + 4x_2 \end{cases}$$

6. Find the general solution to the following first order system:

$$\begin{cases} x_1' = x_1 - 5x_2 \\ x_2' = x_1 - x_2 \end{cases}$$

7. Using the definition of the Laplace transform, compute $\mathscr{L}\{f(t)\}$, where

$$f(t) = \begin{cases} t^2 & \text{for } 0 \le t \le 1 \\ 1 & \text{for } t > 1 \end{cases}$$

8. Using the definition of the Laplace transform, compute $\mathscr{L}\{f(t)\}$, where

$$f(t) = \begin{cases} e^t & \text{for } 0 \le t \le 3 \\ 4 & \text{for } t > 3 \end{cases}$$

9. It is known, for "nice" enough matrices, that we can define the trigonometric functions of a matrix in the following way

$$\sin(tA) = \sum_{n=0}^{\infty}(-1)^n \frac{t^{2n+1}A^{2n+1}}{(2n+1)!}, \quad \cos(tA) = \sum_{n=0}^{\infty}(-1)^n \frac{t^{2n}A^{2n}}{(2n)!}$$

Show that

$$\frac{d}{dt}\left(\sin(tA)\right) = A\cos(tA)$$