Name:_____

Math 3113 Section 003 *Practice Exam 1* *September 4, 2014*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Consider the following IVP:

$$\begin{cases} y' = \sqrt{y-5} \\ y(a) = b \end{cases}$$

Use the theorem of existence to determine for what values of b there are: infinitely many solutions, a unique solution and no solution.

2. Consider the following IVP:

$$\begin{cases} y' = \ln(y-3) \\ y(a) = b \end{cases}$$

Use the theorem of existence to determine for what values of b there are: infinitely many solutions, a unique solution and no solution.

3. Solve the following differential equation:

$$x^2y' = 1 - x^2 + y^2 - x^2y^2$$

4. Solve the following differential equation:

$$\frac{dy}{dx} = 3\sqrt{xy}$$

5. Solve the following differential equation:

$$2xyy' = x^2 + 2y^2$$

6. Solve the following differential equation:

$$\frac{xy'}{y} = 4x^2 + \ln y$$

7. Consider the following two functions: $y_1 = x^2$ and $y_2 = x^{-3}$. Determine where y_1 and y_2 are linear independent and show that they both solve the following differential equation:

$$x^2y'' + 2xy' - 6y = 0$$

8. Consider the following two functions: $y_1 = x$ and $y_2 = x \ln x$. Determine where y_1 and y_2 are linear independent and show that they both solve the following differential equation:

$$x^2y'' - xy' + y = 0$$

9. (BONUS) Let $f(x)$ be a real valued differentiable function on all of $\mathbb{R}$. Define $u(x)$ in the following way

$$u(x) = \sin x \int_0^x f(t) \cos t \; dt - \cos x \int_0^x f(t) \sin t \; dt$$

Verify that u is a solution to the following differential equation:

$$\frac{d^2u}{dx^2} + u = f(x)$$