

Name: _____

Math 3113

Practice Exam 1

September 3, 2012

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Let C_1 and C_2 be constants. Let

$$y = C_1 (x - \ln x) + C_2 \left(\frac{1}{x} - \ln x \right)$$

Verify that y is a solution to the following differential equation:

$$x^2 y'' + xy' - y = \ln x$$

2. Let C_1 and C_2 be constants. Let

$$y = C_1 (\cos x - \cos 2x) + C_2 (\sin x - \cos 2x)$$

Verify that y is a solution to the following differential equation:

$$y'' + y = 3 \cos 2x$$

3. Given the following IVP:

$$\begin{cases} y' = x \ln y \\ y(1) = 1 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

4. Given the following IVP:

$$\begin{cases} y' = x^2 - y^2 \\ y(0) = 1 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

5. Given the following IVP:

$$\begin{cases} y' = 1 + x^2 + y^2 \\ y(0) = 2 \end{cases},$$

use the theorem of existence to determine if a solution exists and if it's unique. Also draw the directional field and draw the solution curve to the IVP if it exists.

6. Solve the following differential equation:

$$x^2 y' = 1 - x^2 + y^2 - x^2 y^2$$

7. Solve the following differential equation:

$$y' \tan x = y$$

8. Solve the following differential equation:

$$y' + y'x^2 = (1 + y)^2$$

9. Solve the following IVP:

$$\begin{cases} x \frac{dy}{dx} = 2y + x^3 \cos x \\ y\left(\frac{\pi}{2}\right) = \pi^2 \end{cases}$$

10. Solve the following IVP:

$$\begin{cases} (x^2 + 4)y' + 3xy = x \\ y(0) = 1 \end{cases}$$

11. Solve the following IVP:

$$\begin{cases} y' - 2y = 3e^{2x} \\ y(0) = 0 \end{cases}$$

12. Solve the following differential equation:

$$xy' + e^y y' = xe^{-y} - 1$$

13. Solve the following differential equation:

$$3y^2y' = 3x^4 + y^3$$

14. Solve the following differential equation:

$$xyy' = y^2 + x\sqrt{4x^2 + y^2}$$

15. Let C be a constant. Define $u(x)$ in the following way

$$u(x) = Ce^{x^2} + \int_0^x e^{x^2-t^2} \cosh t \, dt$$

Verify that u is a solution to the following differential equation:

$$u' - 2xu = \cosh x$$