

1. $y(x) = \frac{C + \sin x}{1 + x}$
2. $y(x) = 2 \exp\left(-\frac{3}{2}x^2\right) + C \exp\left(-\frac{3}{2}x^2\right) (x^2 + 1)^{\frac{3}{2}}$
3. $y(x) = x^2 \sin x + Cx^2$
4. $y(x) = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{4}x \sinh(2x) - \frac{1}{16} \sinh(2x)$
5. $y(x) = C_1 \cos x + C_2 \sin x + \cos x \ln |\csc x + \cot x| - 1$
6. $y(x) = C_1 \cos x + C_2 \sin x + \cos x \ln |\sec x + \tan x| - \cos^2 x - \sin(2x)$
7. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} -\sin(3t) \\ \cos(3t) \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} \cos(3t) \\ \sin(3t) \end{pmatrix} e^{4t}$
8. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{5t} + C_2 \begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{-2t}$
9. $\begin{pmatrix} x \\ y \end{pmatrix} = C_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

10. All the eigenvalues, λ are $\lambda \geq 1$. For the eigenvalue $\lambda = 1$, the corresponding eigenfunction is $y(x) = xe^{-x}$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \alpha_n^2$ where α_n is a root to the following equation $\tan \alpha = \alpha$. The corresponding eigenfunctions are $y_n(x) = e^{-x} \sin(\alpha_n x)$ for $\lambda > 1$.

11. All the eigenvalues, λ are $\lambda > 1$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \pi^2 n^2$ where n is an integer. The corresponding eigenfunctions are $y_n(x) = e^{-x} \sin(\pi n x)$.

12. All the eigenvalues, λ are positive. When $\lambda > 0$ one has $\lambda_n = \alpha_n^2$ where α_n is a root to the following equation $\tan \alpha = -\alpha$. The corresponding eigenfunctions are $y_n(x) = \sin(\alpha_n x)$.

13. $x(t) = -\frac{1}{3} \cos t + \frac{1}{3} \cos(3t)$
14. $x(t) = \frac{1}{8} \cos t + \sin t - \frac{1}{8} \cos(3t)$
15. $x(t) = \frac{1}{9} e^{-t} - \frac{1}{9} e^{-2t} \cos(2t) + \frac{1}{18} e^{-2t} \sin(2t)$
16. $x(t) = \frac{C}{2} (\sin t - t \cos t)$
17. $x(t) = \frac{C}{6} t^3 e^{-t}$

$$\mathbf{18.} \quad x(t) = \frac{C}{2} t^2 e^{-3t}$$

$$\mathbf{19.} \quad x(t) = \frac{1}{2} \sin(2t)$$

$$\mathbf{20.} \quad x(t) = \frac{1}{2} \sin(2t) + \frac{1}{2} \sin(2t - \pi) u(t - \pi)$$

$$\mathbf{21.} \quad \mathcal{L}(1 + [[t]]) = \frac{1}{s(1 - e^{-s})}$$