Name:_____

Math 3113 *Practice Final Exam* *November 21, 2012*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Solve the following differential equation:

$$(1+x)y' + y = \cos x$$

2. Solve the following differential equation:

$$(x^2+1)y' + 3x^3y = 6x \exp\left(-\frac{3}{2}x^2\right)$$

3. Solve the following differential equation:

$$xy' = 2y + x^3 \cos x$$

4. Solve the following differential equation:

$$y'' - 4y = \sinh(2x)$$

5. Solve the following differential equation:

$$y'' + y = \csc^2 x$$

6. Solve the following differential equation:

$$y'' + y = \tan x$$

7. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 - 3x_2 \\ x_2' = 3x_1 + 4x_2 \end{cases}$$

8. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 4x_1 + 2x_2 \\ x_2' = 3x_1 - x_2 \end{cases}$$

9. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 2x_1 + 3x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$$

10. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = 0, \quad y'(1) = 0 \end{cases}$$

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$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = 0, \quad y(1) = 0 \end{cases}$$

12. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0, \quad y(1) + y'(1) = 0 \end{cases}$$

13. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \cos t \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

14. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + x = \cos(3t) \\ x(0) = 1, \quad x'(0) = 0 \end{cases}$$

15. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x' + 8x = e^{-t} \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

16. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' - 2x' + tx = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

Hint: $\sin\tau \sin(t-\tau) = \dfrac{1}{2}\left(\cos(2\tau - t) - \cos t\right)$

17. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + (t-2)x' + x = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

18. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} tx'' + (3t-1)x' + 3x = 0 \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

19. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \delta(t) \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

20. Use the Laplace transform to solve the following initial value problem:

$$\begin{cases} x'' + 4x = \delta(t) + \delta(t-\pi) \\ x(0) = 0, \quad x'(0) = 0 \end{cases}$$

21. Compute the Laplace transform of the following function:

$$f(t) = 1 + [[t]]$$

on $[0, \infty)$, where $[[t]]$ is the greatest integer function.