1. $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{-2t}$

2. $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} \cos(2t) - 2\sin(2t) \\ -\frac{11}{9}\cos(2t) \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} \sin(2t) \\ -\frac{4}{9}\sin(2t) + \frac{2}{9}\cos(2t) \end{pmatrix} e^{-3t}$

3. $\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{2t}$

4. $\det(A) = e^t \ln t - t^3 - 2t$, $\det(B) = t\sin^2 t - \cos t \ln t$. $\det(B) \neq 0$ when $t > 0$ so $B^{-1} = \dfrac{1}{t\sin^2 t - \cos t \ln t} \begin{pmatrix} t\sin t & -\cos t \\ -\ln t & \sin t \end{pmatrix}$ Finally

$$(AB)' = \begin{pmatrix} \frac{1}{t}\sin t + \ln t & \frac{1}{t}\cos t + t\sin t \\ 2t\sin t + e^t \ln t & 2t\cos t + te^t \sin t \end{pmatrix}$$
$$+ \begin{pmatrix} \ln t \cos t + 1 & -\sin t \ln t + t\sin t + t^2 \cos t \\ (t^2+2)\cos t + \frac{1}{t}e^t & -(t^2+2)\sin t + e^t \sin t + te^t \cos t \end{pmatrix}$$

5. $\det(A) = t^3 + t^2 - 2t - e^t e^{\sin t}$, $\det(B) = t\ln t \cos t - e^t \sin t$. $\det(B) \neq 0$ when $t > 0$ so $B^{-1} = \dfrac{1}{t\ln t\cos t - e^t \sin t} \begin{pmatrix} \cos t & -\sin t \\ -e^t & t\ln t \end{pmatrix}$ Finally

$$(AB)' = \begin{pmatrix} (2t^2+t)\ln t + e^{2t} & (2t+1)\sin t + e^t \cos t \\ te^{\sin t} \ln t \cos t + e^t & e^{\sin t}\cos t \sin t + \cos t \end{pmatrix}$$
$$+ \begin{pmatrix} (t^2 - t - 2)(\ln t + 1) + e^{2t} & (t^2 + t - 2)\cos t - e^t \sin t \\ (\ln t + 1)e^{\sin t} + te^t & e^{\sin t}\cos t - t\sin t \end{pmatrix}$$

6. $x(t) = c_1 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

7. $x(t) = c_1 \begin{pmatrix} 7\cos(\sqrt{11}t) - \sqrt{11}\sin(\sqrt{11}t) \\ -6\cos(\sqrt{11}t) \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} \sqrt{11}\cos(\sqrt{11}t) + 7\sin(\sqrt{11}t) \\ -6\sin(\sqrt{11}t) \end{pmatrix} e^{2t}$

8. $x(t) = c_1 \begin{pmatrix} \cos(4t) - 2\sin(4t) \\ 2\cos(4t) \end{pmatrix} e^{5t} + c_2 \begin{pmatrix} 2\cos(4t) + \sin(4t) \\ 2\sin(4t) \end{pmatrix} e^{5t}$

9. All the eigenvalues, λ are $\lambda \geq 1$. For the eigenvalue $\lambda = 1$, the corresponding eigenfunction is $y(x) = (1-x)e^{-x}$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \pi^2 n^2$ where n is an integer. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\pi n x)$.

10. All the eigenvalues, λ are $\lambda > 1$. For eigenvalues $\lambda > 1$ one has $\lambda_n = 1 + \alpha_n^2$ where α_n is a root to the following equation $\tan\alpha = \alpha$. The corresponding eigenfunctions are $y_n(x) = e^{-x}\sin(\alpha_n x)$.

11. All the eigenvalues, λ are positive. When $\lambda > 0$ one has $\lambda_n = \alpha_n^2$ where α_n is a root to the following equation $\tan\alpha = -\alpha$. The corresponding eigenfunctions are $y_n(x) = \sin(\alpha_n x)$.

12. One computes L_1L_2x and L_2L_1x to get

$$L_1L_2x = a_1a_2x^{(4)} + (b_1a_2 + a_1b_2)x''' + (c_1a_2 + b_1b_2 + a_1c_2)x'' + (c_1b_2 + b_1c_2)x' + c_1c_2x$$
$$L_2L_1x = a_2a_1x^{(4)} + (b_2a_1 + a_2b_1)x''' + (c_2a_1 + b_2b_1 + a_2c_1)x'' + (c_2b_1 + b_2c_1)x' + c_2c_1x$$

Then one compares the coefficients to see that $L_1L_2x = L_2L_1x$.