Name:_____

Math 3113 *Practice Exam 3* *October 30, 2012*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Find the general solution to the following first order system:

$$\begin{cases} x' = 3x - y \\ y' = 5x - 3y \end{cases}$$

2. Find the general solution to the following first order system:

$$\begin{cases} x' = x + 9y \\ y' = -2x - 5y \end{cases}$$

3. Find the general solution to the following first order system:

$$\begin{cases} x' = 2x + y \\ y' = x + 2y - e^{2t} \end{cases}$$

4. Let

$$A(t) = \begin{pmatrix} \ln t & t \\ t^2 + 2 & e^t \end{pmatrix} \text{ and } B(t) = \begin{pmatrix} \sin t & \cos t \\ \ln t & t \sin t \end{pmatrix}$$

Compute $\det(A)$, $\det(B)$, and B^{-1} where it exists. Finally compute $\dfrac{d}{dt}(AB)$.

5. Let

$$A(t) = \begin{pmatrix} t^2 + t - 2 & e^t \\ e^{\sin t} & t \end{pmatrix} \text{ and } B(t) = \begin{pmatrix} t \ln t & \sin t \\ e^t & \cos t \end{pmatrix}$$

Compute $\det(A)$, $\det(B)$, and B^{-1} where it exists. Finally compute $\dfrac{d}{dt}(AB)$.

6. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 2x_1 + 3x_2 \\ x_2' = 2x_1 + x_2 \end{cases}$$

7. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 9x_1 + 5x_2 \\ x_2' = -6x_1 - 2x_2 \end{cases}$$

8. Find the general solution to the following first order system:

$$\begin{cases} x_1' = 7x_1 - 5x_2 \\ x_2' = 4x_1 + 3x_2 \end{cases}$$

9. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = y(1) = 0 \end{cases}$$

10. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + 2y' + \lambda y = 0 \\ y(0) = y'(1) = 0 \end{cases}$$

11. Find all the eigenvalues and eigenfunctions of the following boundary value problem:

$$\begin{cases} y'' + \lambda y = 0 \\ y(0) = 0, \quad y(1) + y'(1) = 0 \end{cases}$$

12. Suppose $x(t)$ is a differentiable function with a_1, a_2, b_1, b_2, c_1, and c_2 are constants. Let

$$L_1x = a_1D^2x + b_1Dx + c_1x \text{ and } L_2x = a_2D^2x + b_2Dx + c_2x.$$

Show that

$$L_1L_2x = L_2L_1x.$$