Name:________________________________

Math 2924 Section 170 — *Practice Exam 2* — *March 9, 2015*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

1. Determine if the following integral converges or diverges:

$$\int_{-\infty}^{\infty} x^3 e^{-x^4}\, dx$$

2. Determine if the following integral converges or diverges:

$$\int_{0}^{\infty} \frac{e^x}{e^{2x}+3}\, dx$$

3. Set up an integral the represents the length of the curve $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$. (Do NOT evaluate the integral)

4. Set up an integral the represents the length of the curve $y = 3 + \dfrac{1}{2}\cosh(2x)$ on $0 \leq x \leq 1$. (Do NOT evaluate the integral)

5. Find the sum of the following series:

$$\sum_{n=1}^{\infty} \left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}} \right)$$

6. Find the sum of the following series:

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

For problems 7 - 12, determine if the following series converge or diverge:

7. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

9. $\displaystyle\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^2 e^{-n}$

10. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$

11. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\cos\left(\frac{\pi}{n}\right)$

12. $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)\sin\left(\frac{\pi}{n}\right)$

13. Let f be a differentiable function on $(0, 1)$. Show that

$$\lim_{n\to\infty} \int_0^1 f(x)\cos(nx)\ dx = 0$$

(Hint: You will need to integrate by parts and use the fact that f' is continuous on $(0, 1)$ and hence it's bounded on $(0, 1)$.)