Name:_____

Math 2443 Section 270 *Practice Exam 2* *June 20, 2013*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 - 6, compute the following double integrals:

1. $\displaystyle\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)\, dx\, dy$

2. $\displaystyle\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos x \sqrt{1+\cos^2 x}\, dx\, dy$

3. $\int\int_R \frac{y^2}{x^2+y^2}\, dA$ where $R = \{(x,y) : 1 \leq x^2 + y^2 \leq 4\}$.

4. $\int\int_D \cos\sqrt{x^2+y^2}\, dA$ where D is the disk with center the origin and radius 2.

5. $\int\int_D x^2 y\, dA$ where D is the top half of the disk with center the origin and radius 5.

6. $\int\int_D x \; dA$ where D is the region in the first quadrant that lies between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$.

For problems 7 - 12, compute the following triple integrals:

7. $\int\int\int_E \frac{z}{x^2 + z^2} \; dV$ where $E = \{(x, y, z) : 1 \leq y \leq 4, y \leq z \leq 4, 0 \leq x \leq z\}$.

8. $\int\int\int_E y \; dV$ where $E = \{(x, y, z) : 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}$.

9. $\int\int\int_E \sqrt{x^2+y^2}\, dV$ where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$.

10. $\int\int\int_E (x+y+z)\, dV$ where E is the solid in the first octant that lies under the paraboloid $z=4-x^2-y^2$.

11. $\int\int\int_E (x^2+y^2+z^2)^2\, dV$ where E is the ball with center the origin and radius 5.

12. $\int\int\int_E xe^{x^2+y^2+z^2}\ dV$ where E is the portion of the unit ball $x^2+y^2+z^2 \leq 1$ that lies in the first octant.

13. The following integral is an improper integral on the cube $[0,1]\times[0,1]\times[0,1]$

$$\int_0^1\int_0^1\int_0^1 \frac{1}{1-xyz}\ dx\ dy\ dz$$

Show that one has the following:

$$\int_0^1\int_0^1\int_0^1 \frac{1}{1-xyz}\ dx\ dy\ dz = \sum_{n=1}^{\infty}\frac{1}{n^3}$$