Name:______________________________

Math 2443 Section 270 *Practice Exam 1* *June 20, 2013*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 and 2, determine if the following limit exists. If it does compute it, otherwise show it does not exist.

1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - y^4}{x^2 + y^2}$

2. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$

3. Find the equation of the tangent plane for $z = x\sin(x+y)$ at the point $(-1, 1, 0)$.

4. Find the equation of the tangent plane for $z = \ln(x - 2y)$ at the point $(3, 1, 0)$.

5. Let $P = \sqrt{u^2 + v^2 + w^2}$, $u = xe^y$, $v = ye^x$, and $w = e^{xy}$. Compute the following partial derivatives:

$$\frac{\partial p}{\partial x} \quad \text{and} \quad \frac{\partial p}{\partial y}$$

6. Let $N = \dfrac{p+q}{p+r}$, $p = u + vw$, $q = v + uw$, and $r = w + uv$. Compute the following partial derivatives:

$$\frac{\partial N}{\partial u}, \quad \frac{\partial N}{\partial v}, \quad \text{and} \quad \frac{\partial N}{\partial w}$$

7. If $z = f(x, y)$, $x = g(t)$, $y = h(t)$, $g(4) = 5$, $h(4) = 12$, $g'(4) = 9$, $h'(4) = 7$, $f_x(5, 12) = 8$, and $f_y(5, 12) = -7$, compute:

$$\left.\frac{dz}{dt}\right|_{t=4}$$

8. If $z = f(x, y)$, $x = g(t)$, $y = h(t)$, $g(\pi) = 4$, $h(\pi) = e$, $g'(\pi) = 10$, $h'(\pi) = 23$, $f_x(4, e) = 2$, and $f_y(4, e) = 0$, compute:

$$\left.\frac{dz}{dt}\right|_{t=\pi}$$

9. Compute the directional derivative of $f(x, y) = e^x \sin y$ at the point $(0, \pi/3)$ in the direction of $\mathbf{v} = -6\mathbf{i} + 8\mathbf{j}$.

10. Compute the directional derivative of $g(s, t) = te^{st}$ at the point $(0, 2)$ in the direction of $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$.

11. Use Lagrange multipliers to find the max and min values of $f(x, y) = x^2 + y^2$ subject to $xy = 1$.

12. Use Lagrange multipliers to find the max and min values of $f(x, y) = y^2 - x^2$ subject to $x^2 + 4y^2 = 4$.

13. If $z = f(t)$, where $t = x - y$ show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$