

Name: \_\_\_\_\_

Math 221 Section 10336

*Practice Final Exam*

*June 11, 2012*

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Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work or answer, you will receive little or no credit!

For problems 1 and 2, compute the following limits if they exist.

1.  $\lim_{x \rightarrow 3} \frac{2x^2 - 3x - 9}{x - 3}$

2.  $\lim_{x \rightarrow \infty} \frac{5x^3 + 3x^2 + ex - 1}{\pi x^3 - 5x + 7e}$

For problems 3 - 8, differentiate each function.

3.  $g(x) = \frac{\sin^{-1} x}{5x^2 + 7}$

4.  $f(x) = \frac{e^x \cos x}{x^2 - 2}$

5.  $y = x^7 \sin(\ln x)$

6.  $g(x) = e^{\sin x} \cos x$

7.  $h(t) = e^{t^2} \ln t$

8.  $r(x) = \ln(\cos x)e^{2x}$

For problems 9 and 10, compute the derivative of the following functions using the definition of the derivative (4-step process).

9.  $f(x) = 2x^2 + x - 1$

10.  $f(x) = x^3 - 2$

For problems 11 - 16, perform the indicated integration.

11.  $\int x^2 \sqrt[7]{3x^3 + 8} \, dx$

12.  $\int x^4 (x^5 + 7)^\pi \, dx$

13.  $\int x\sqrt{x+3} \, dx$

14.  $\int x\sqrt[3]{x-7} \, dx$

15.  $\int_0^1 \frac{3x^2 \, dx}{\sqrt[3]{x^3+7}}$

16.  $\int_{-1}^0 (x^2 + 2x + 3)^\pi (x + 1) dx$

For problems 17 and 18, differentiate the following functions.

17.  $g(x) = \int_1^{\sec x} \frac{\cos t}{\sqrt{1 + \cos^{-1} t}} dt$

18.  $f(x) = \int_x^{e^x} \ln \left( \frac{\sin t}{t^2 + 7} - \sin^{-1}(t^2 - 2) \right) dt$

For problems 19 and 20, find the general antiderivative of the following functions.

19.  $f(x) = \frac{1}{x\sqrt[3]{x}} - 3x^2 + 7x^4 + \frac{1}{x}$

20.  $f(x) = \frac{x + \sqrt{x}}{x^2} - 2x^3 + x$

For problems 21 and 22, find the area of the region bounded by the given curves.

21. The curves  $y = x^2 + 1$ ,  $y = x$ ,  $x = 0$ , and  $x = 1$ .

**22.** The curves  $y = x + 1$ ,  $y = 9 - x^2$ ,  $x = -1$ , and  $x = 2$ .

For problems 23 - 26, set up an integral that represents the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

**23.** Between the curves  $y = x$  and  $y = x^2$  and about the line  $y = 2$ .

**24.** Between the curves  $y = 1 - x^2$  and  $y = 0$  and about the  $x$ -axis.

25. Between the curves  $y = x - x^2$  and  $y = 0$  and about the line  $x = 2$ .

26. Between the curves  $y = 3 + 2x - x^2$  and  $x + y = 3$  and about the  $y$ -axis.

For problems 27 - 30, determine whether the following improper integrals converge or diverge. If they converge, evaluate them.

27. 
$$\int_1^{\infty} \frac{1}{(3x + 1)^2} dx$$

28.  $\int_0^{\infty} \frac{x}{\sqrt{x^2+1}} dx$

29.  $\int_0^1 \frac{1}{\sqrt{x}} dx$

30.  $\int_2^5 \frac{1}{(x-3)^2} dx$

For problems 31 and 32, use the definition of the integral to evaluate the following definite integrals.

**31.**  $\int_0^2 x \, dx$

**32.**  $\int_0^1 x^3 \, dx$